MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXXI. DECEMBER 10, 1920. No. 2

Professor A. FOWLER, F.R.S., PRESIDENT, in the Chair.

Thomas Allison, Barrister and Solicitor of the Supreme Court of New Zealand, 54 Glasgow Street, Wanganui, N.Z.; and
William Henry Connell, Engineers' Manager, T. Cooke & Sons, York, and Torver House, Stockton Lane, York,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Cumberland Clark, F.R.G.S., F.Z.S., 22 Kensington Park Gardens, London, W. 11 (proposed by P. E. Vizard);
Robert Cochran Gray, M.A., B.Sc., University, Perth, Western Australia (proposed by J. Jackson);
Masao Hasimoto, International Latitude Observatory, Mizusawa, Japan, and 35 Grantchester Street, Cambridge (proposed by A. S. Eddington);
George Fredric Sowden Hills, B.A. (Camb.), Aston Grays, West End Avenue, Pinner (proposed by A. S. Eddington);
Daniel James Jones, City Observatory, Cardiff, and 101 Mackintosh Place, Roath Park, Cardiff (proposed by A. Fowler);
Edward Arthur Milne, B.A., Fellow of Trinity College, and Assistant Director of the Solar Physics Observatory, Cambridge (proposed by H. F. Newall);
George Morham, Fellow of the Faculty of Surveyors, 17 Mansionhouse Road, Edinburgh (proposed by W. Peck);

Cyril Potter, Dental Surgeon, 7 Balfour Road, Birkenhead (proposed by W. H. Steavenson);

Charles Bousfield Shaw, Member of the Bar, Secretary and Treasurer, Royal Agricultural Benevolent Institution, 8 Milborne Grove, The Boltons, London, S.W. 10 (proposed by J. Evershed); and

A. R. Stoney Smith, White Waltham Grove, near Maidenhead (proposed by R. A. Fisher).

Forty-two presents were announced as having been received since the last Meeting.

Mr. P. J. Melotte and Dr. W. H. Steavenson were appointed Auditors of the Treasurer's accounts for 1920.

A Solution of Ancient Eclipses of the Sun. By J. K. Fotheringham, M.A., D.Litt.

In *M.N.*, **80**, 578–81, I gave the results derived from the times and magnitudes of ancient lunar eclipses, ancient occultations of stars and conjunctions of the Moon with stars, equinox observations, and observations of solar eclipses, for the determination of the accelerations of the Sun and Moon. In the case of the solar eclipses, the result given was based, not on a full discussion, but on a tentative examination made by Herr Schoch of Heidelberg of the effect of introducing into the motions of the Sun and Moon respectively certain corrections which I had suggested in earlier papers. The present paper contains the results of an analysis of those solar eclipses which would appear either from the passages in which they are recorded, or from the investigations of earlier inquirers, to be of value for the determination of the two accelerations.

The eclipses selected for computation are:—

(1) The eclipse of Babylon, −1062 July 31.
(2) The eponym canon eclipse, −762 June 15.
(3) The eclipse of Archilochus, −647 April 6.
(4) The eclipse of Thales, −584 May 28.
(5) The eclipse of Pindar, −462 April 30.
(6) The eclipse of Thucydides, −430 August 3.
(7) The eclipse of Agathocles, −309 August 15.
(8) The eclipse of Hipparchus, −128 November 20.
(9) The eclipse of Phlegon, +29 November 24.
(10) The eclipse of Plutarch, +71 March 20.
(11) The eclipse of Theon, +364 June 16.

In each case the day specified is reckoned from midnight.

Other eclipses have been omitted because there is no sufficiently strong presumption as to the magnitude of the eclipse at any particular place. The so-called eclipse of Larissa, which has been variously dated −556 May 19 and −602 May 18, is rejected because the record,* whether true or false, seems clearly to refer to some phenomenon other than an eclipse.

(1) *The eclipse of Babylon.*—L. W. King, *Chronicles concerning Early Babylonian Kings* (1907), 2, 76: "On the twenty-sixth day of the month Sivan in the seventh year the day was turned to night, and fire in the midst of heaven . . ." The passage is discussed at length by King in the first volume of the same work, pp. 232–240. The eclipse was identified by Dr. Cowell in *M.N.*, **65** (1905), 861–7. The text to which the passage belongs is mainly occupied with portents observed at Babylon, such as wild animals or floods entering the city. There are, however, a few references to other places. Thus mention is made of the Lady of Nineveh, the river Tigris, Bit-Ursag in the district of Nippur, and the city of Kar-bel-matati.† As a rule not more than two or three portents are recorded in one year, and the presumption is that the phenomenon described is a rare one, was observed at Babylon, and may reasonably have been a total eclipse of the Sun. King argues against the identification of the portent with a thunderstorm or dust-storm, and discounts the difference between the phrase "the day was turned to night" and the technical phrase "heaven-darkening," which was from at least the eighth century B.C. used for an eclipse. The portent would appear to have been observed somewhere in the eleventh century B.C., and the eclipse of −1062 July 31 is the only one which can have been total at Babylon within that century. King has a difficulty about the date, because the identification implies that Sivan was falling a month late; but intercalation at Babylon was notoriously irregular, and a latitude of one month in the date of a lunar month is not surprising. A further difficulty, which King does not notice, is that the portent occurred on the 26th day of the month. Unless the Babylonian calendar was in disorder at the time, this is too early for a solar eclipse. The Babylonian months were supposed to begin with the first appearance of the lunar crescent, and on this system −1062 July 31 should have been not the 26th but the 28th day of the month. Herr Schoch has suggested, in an unpublished memoir of which he has kindly sent me a copy, that "26th" is a mistake of the scribe for "28th." Dr. Langdon has kindly verified the reading "26th" on the tablet at the British Museum, but he assures me that the cuneiform symbols for the two numbers are so

* Xenophon, *Anabasis*, 3, 4, 8: "This city [*i.e.* Larissa] the king of the Persians besieged, what time the Persians strove to snatch their empire from the Medes, but he could in no wise take it; then a cloud hid the face of the sun and blotted out the light thereof, until the inhabitants were gone out of the city, and so it was taken." Xenophon, *March of the Ten Thousand*, tr. by Dakyns (1901), 86.

† Dr. Langdon assures me that Kar-bel-matati was a city in the extreme south of Babylonia.

similar that "26th" might easily have been substituted by an ancient scribe for "28th." In fact, he has gone the length of adopting the identification of this portent with the eclipse of −1062 July 31 as a starting-point for the construction of Babylonian chronology.

In *Zeitschrift für Assyriologie*, 27 (1912), 388–90, Dr. Weidner rejects the interpretation of the phenomenon as an eclipse on the ground that the technical phrase for an eclipse is not used. He finds references to fire in heaven on a mutilated tablet, on the other side of which there are references to the morning and evening glow of the clouds. He somewhat wildly infers from this that "fire in the midst of heaven," which Dr. Cowell had suggested might mean the corona, refers to a red glow on the clouds, and interprets the whole phenomenon as a thunderstorm. There is, of course, no evidence that the two sides of the tablet which he cites are dealing with cognate subjects.

Father Kugler has kindly answered an inquiry of mine about the phenomenon in question. While rejecting Dr. Weidner's interpretation, he holds that the phenomenon must have been meteorological, not astronomical, because, as he has shown in *Zeitschrift der deutschen morgenländischen Gesellschaft*, 56 (1902), 60–70, the technical term for an eclipse is regularly accompanied by the name of the luminary where an astronomical eclipse is intended, and is constantly used without such a name to indicate a darkness due to meteorological causes. He also urges against the interpretation of the phenomenon as an eclipse that it is recorded as falling on the 26th day of the month. In his opinion the reference is to a shower of meteoric dust, accompanied by glowing meteorites. For examples of such showers he refers to Arago's posthumously published *Astronomie populaire*, 4 (1857), 209–16,* and in particular to the showers of 472 November 5 or 6, 1792 August 27–29, 1814 July 3 and 4, 1819 November. On the last of these occasions Arago asserts that there was an extraordinary darkness of the sky accompanied by fiery apparitions. He cites *Annales de chemie et de physique*, 15 (1820), apparently pp. 422, 426, 427, but only a part of the phenomena which he describes is recorded there, and in particular there is no mention of fiery apparitions, though there is of black rain. It is probable that Arago relied in part on some source to which he left no reference.

Dr. Langdon agrees that "fire in the midst of heaven" should, according to Babylonian usage, be a meteor, but attaches no importance to the fact that the phrase "the day was turned to night," which would excellently describe a total eclipse of the Sun, is not the correct astronomical phrase. He lays stress on the fact that the text in which the passage occurs is not written by an astronomer, but in all probability by a divination scholar; and of course it must be remembered that it is older than those by means of which Father Kugler defines the established usage. It is

* This passage is omitted from the English translation, edited by Smyth and Grant.

certainly remarkable that there should have been an eclipse which may, consistently with other evidence, have been total at or near Babylon at or near the date to which the text refers, and that it should have fallen at the right time of year. I take it accordingly that there is a presumption, but not a certainty, that the eclipse of −1062 July 31 was total at Babylon, for which I take λ (east longitude) = 44°·50, φ (north latitude) = 32°·52.

(2) *The eponym canon eclipse.**—The Assyrian Chronicle translated by Dr. Sayce in *Records of the Past*, new series, 2 (1890), 124: "Isid-Raki's-rabe [or rather Ishdi-Sagale †] of the city of Gozan. Insurrection in the city of Assur. In the month Sivan the sun was eclipsed." The date is beyond question, since the Assyrian eponym canon overlaps Ptolemy's Canon of Kings, and the year −762 is obtained by counting backwards to the year of Ishdi-Sagale. The Assyrian Chronicle is a fuller form of the eponym canon, containing not merely lists of eponyms, but striking events from all parts of Assyria, and Assyrian wars beyond Assyria. The reference to Sivan gives the time of year, and the identification with the eclipse of June 15 in that year, first made by George Smith in 1867,‡ is certain. As the eclipse is the only eclipse mentioned in this Chronicle, which covers an interval of 155 years, there can be no reasonable doubt that it had been reported as a total eclipse. On the other hand, there is no reason why the eclipse should have been total either at Nineveh, where the extant copies of the Chronicle were discovered, or at Calah, which was the Assyrian capital at the date of the eclipse, and where this portion of the Chronicle was probably put together. I have considered it sufficient that the eclipse should have been total somewhere in Assyria, and have taken as the limits between which totality must have occurred, λ42°·19, φ37°·40 and λ45°·15, φ 34°·18.

(3) *The eclipse of Archilochus.*—Archilochus, fr. 74, in Bergk, *Poetæ Elegiaci et Iambographi* (1915): "Nothing there is beyond hope, nothing that can be sworn impossible, nothing wonderful, since Zeus father of the Olympians made night from midday, hiding the light of the shining sun, and sore fear came upon men."

The eclipse was identified by Oppolzer.§ The description clearly refers to a total eclipse, and there is a presumption that Archilochus saw it. As the life of Archilochus was divided between Paros and Thasos, with occasional travels elsewhere, the presumption is that the eclipse was total either at Paros or at Thasos. I find by computation that the width of the belt of totality is practically equal to the distance between these islands, and I have therefore assumed that the eclipse was total somewhere

* For the literature down to 1899 on this and the remaining eclipses, with the exception of (5) and (8), see Professor Ginzel's *Spezieller Kanon der Sonnen- und Mondfinsternisse* (1899).

† Tallqvist, *Assyrian Personal Names* (1914), 104.

‡ See his *Assyrian Discoveries* (1875), pp. 12, 13.

§ *Sitzungsberichte der k. Akad. der Wissenschaften, Vienna, Math.-nat. Classe*, 86 (1882), 790–3.

between the limits λ24°·56, φ40°·78 and λ25°·25, φ37°·01, and have not thought it necessary to exclude any part of the intervening space. The computed time of the eclipse falls about ten in the morning instead of at midday. But a tendency to transfer eclipses to midday will be found again in the cases of the eclipses of Phlegon and Plutarch.

(4) *The eclipse of Thales.*—Herodotus, 1, 74, tr. by Rawlinson (1909), 1, 36, 37: "War broke out between the Lydians and the Medes, and continued for five years, with various success. In the course of it the Medes gained many victories over the Lydians, and the Lydians also gained many victories over the Medes. Among their other battles there was one night engagement. As, however, the balance had not inclined in favour of either nation, another combat took place in the sixth year, in the course of which, just as the battle was growing warm, day was on a sudden changed into night. This event had been foretold by Thales, the Milesian, who forewarned the Ionians of it, fixing for it the very year in which it actually took place. The Medes and Lydians, when they observed the change, ceased fighting, and were alike anxious to have terms of peace agreed on."

An incomplete collection of later references to the eclipse is given by Professor Diels in *Fragmente der Vorsokratiker*, 1 (1912), pp. 8, 9. The war fell, according to Herodotus and several later writers, in the reigns of Alyattes of Lydia and Cyaxares of Media. Cicero, *de Div.*, 1, 49, 112, places the eclipse in the reign of Astyages, the successor of Cyaxares. Eusebius, who separates the eclipse from the war, places both in the reign of Astyages; and Solinus, ed. Mommsen, 85, 9, places the war in the reign of Astyages. Several ancient writers give the date of the eclipse, generally within a few years of −584. Eusebius in his *Chronica* alone gives that precise year. It must be remembered that the eras used by later writers were not adopted till long after the time of Thales, and it was therefore easy for a small error to slip into the conversion of a date. The eclipse was identified with that of −584 May 28 by Bünting, *Chronologia* (1590), 82. But there was formerly a difficulty in the way of accepting this date, because Herodotus appeared to make Astyages die in −557, after a reign of 35 years, and the date −584 seemed in consequence to be inconsistent with Herodotus's statement that the event occurred in the reign of Cyaxares. It is now known, however, from the annalistic tablet of Cyrus that the reign of Astyages lasted till −549, and his accession ought, therefore, to have been somewhere in the neighbourhood of −584. It is impossible to say whether Cyaxares or Astyages was reigning on May 28 of that year, and there appears to be no longer any reason for calling the date in question.

It has frequently been assumed that the battle described by Herodotus must have been in the neighbourhood of the Halys, presumably because Crœsus crossed the Halys when he made war on Persia. But this is quite fanciful. The battle may have been fought almost anywhere in Asia Minor. I have assumed that the

eclipse was total somewhere between λ27°·50, φ37°·00 and λ38°·50, φ41°·00.

For the method by which Thales predicted this eclipse, see Airy, *M.N.*, 18 (1858), 148, and my paper in the *Journal of Hellenic Studies*, 39 (1919), 181–3.

(5) *The eclipse of Pindar.*—Pindar, Pæan ix., ed. Sandys (1919), p. 546: "Beam of the Sun! O thou that seest afar, what wilt thou be devising? O mother of mine eyes! O star supreme, reft from us (κλεπτόμενον) in the daytime! Why hast thou perplexed the power of man and the way of wisdom, by rushing forth on a darksome track?"

See also Pindar, fr. 142, *ibid.*, p. 576: "God can cause unsullied light to spring out of black night. He can also shroud in a dark cloud of gloom the pure light of day."

The eclipse was identified by Ideler * with that of −462 April 30, which was certainly the largest at Thebes in the time of Pindar. The first of the passages cited is part of a pæan written by Pindar, a Theban, and addressed to the Thebans. The terms used seem to indicate a total eclipse of the Sun, and the second passage, from a dance-song, would appear to refer to the same event. There is a reasonable presumption that the eclipse was total at Thebes, λ23°·32, φ38°·32, though the present tense, κλεπτόμενον, is consistent with incomplete action.

(6) *The eclipse of Thucydides.*—Thucydides, 2, 28, tr. by Smith (1919), pp. 309, 311: "During the same summer at the beginning of a lunar month (the only time, it seems, when such an occurrence is possible) the sun was eclipsed after midday; it assumed the shape of a crescent (μηνοειδής) and became full again, and during the eclipse some stars became visible."

The year of this eclipse is clearly defined by Thucydides, and corresponds to our −430. The eclipse was identified by Bünting, p. 108. Though Thucydides does not name the place of observation, it has generally been assumed that it was Athens, an assumption which accords well with the general perspective of Thucydides' narrative of the events of this year. Mr. Munro has, however,† in order to render it possible for Thucydides to have seen stars, and guided by the narrative of negotiations with Thrace, which follows the record of this eclipse, suggested that the real place of observation was at the Odrysian capital in the neighbourhood of Adrianople. In order to examine the question of the visibility of stars, I have computed the stellar magnitudes and altitudes of all the planets and the altitudes of the brighter fixed stars for Athens for the moment of greatest phase of the eclipse, which I have taken to be $5^h\ 36^m$ in the afternoon, local solar time. Treating the stellar magnitudes as valid for stars in the zenith, I have reduced them to the apparent magnitudes proper to their respective altitudes, using $0^m{\cdot}22$ as the coefficient of the atmospheric absorption of light. The resultant magnitudes are Venus −3·35

* See *Pindari Opera*, ed. Boeckh, 2 (2), (1821), 602

† *Classical Quarterly*, 13 (1919), 127, 128.

Mercury +1·3, Mars +4·05, Saturn +0·55, Vega +0·20, α Centauri barely above horizon, Jupiter and Sirius below horizon. It will be seen from this that Vega, the second brightest star above the horizon, was only $\frac{1}{26}$ as bright as Venus. Now, as a general rule, stars other than Venus are not visible to the naked eye during an eclipse except on the verge of totality. In the case of the eclipse of 1912 April 17, I have found no record of any star but Venus being seen without instrumental aid except where the eclipse was total, although there were numerous observers along the long zone where the eclipse passed through France, Belgium, and Germany just missing totality. The eclipse of −430 was an annular eclipse with a maximum phase of about 11·88 digits or 99 per cent. of the Sun's diameter. It fell, therefore, considerably short of totality, and, in view of the faintness of the stars above the horizon, I find it exceedingly difficult to accept Thucydides' statement that some stars became visible. In all probability Venus was the only star visible, and there should have been no difficulty in seeing Venus, even if the magnitude of the eclipse amounted to 10·5 only. As Thucydides mentions the crescent phase of the eclipse, but not the ring phase, it seems probable that the eclipse was not annular at Athens. This is the only presumption that I venture to make, and even this presumption cannot be pressed.

(7) *The eclipse of Agathocles.*—Diodorus, **20**, 5, tr. by Booth (1700), 659: "The next day there was such an Eclipse of the Sun, that the Stars appear'd every where in the Firmament, and the Day was turn'd into Night." This eclipse was first identified by Petavius, *De Doctrina Temporum* (1627), **1**, 804, 805. Since the year is specified by Diodorus, there can be no doubt of the correctness of the identification. As we learn from the context, the eclipse was observed by the fleet of Agathocles the day after it had set out on a voyage lasting six days and six nights, from Syracuse to Latomiæ on the African coast. The historical evidence is insufficient to determine whether he sailed to the north or to the south of Sicily. In calculating the distance traversed it must be remembered that the time of greatest phase is found to be about $7^h\ 40^m$ in the morning, local solar time. I have assumed that the eclipse must have been total somewhere between the limits λ15°·50, φ38°·42, and λ15°·00, φ36°·58. A glance at the chart in Ginzel's *Spezieller Kanon*, p. 186, showing the zone of totality according to different computations, will make it clear that Agathocles is not likely to have travelled further than these points in a direction perpendicular to the line of central eclipse.*

(8) *The eclipse of Hipparchus.*—Cleomedes, *De Motu Circulari Corporum Cœlestium*, 2, 3: "Once it [*sc.* the Sun] was observed to be totally eclipsed at the Hellespont, when at Alexandria it

* There is a reference to this eclipse in Justin, **22**, 6, but he does not define the magnitude of the eclipse, nor the part of the voyage at which it was observed.

was eclipsed with the exception of one fifth of its own diameter, that is, in appearance [*i.e.* as seen by an observer] with the exception of two digits and a fraction."

Pappus, Commentary on the fifth book of the μεγάλη σύνταξις of Ptolemy*: "For in his first book concerning magnitudes and distances he [*i.e.* Hipparchus] uses the following phenomenon: —an eclipse of the Sun which was actually of the whole Sun in the places round the Hellespont, so that no part of it could be seen at the Moon's edge, while at Alexandria in Egypt the Sun was eclipsed to the extent of about four fifths of his diameter."

No clue is given for the identification of this eclipse apart from the names of Hipparchus and Alexandria, and the magnitudes at the Hellespont and at Alexandria. Herr Schoch in his memoir has discussed the possibility of combining the required phases for the Hellespont and Alexandria, and finds that from −320 to −110 the only eclipses which could be made to give anything approximating to the required combination are the eclipse of −309 August 15, just discussed as that of Agathocles, and the eclipse of −128 November 20, with which Hultsch† has identified the eclipse now in question. No additional eclipse could be brought into the discussion by carrying the investigation back to the foundation of Alexandria in −331. Against the identification of the eclipse of Hipparchus with that of Agathocles Herr Schoch argues that it would have been impossible to judge the magnitude of the eclipse of Agathocles at Alexandria with the naked eye, as that eclipse would reach its greatest phase there about $9^h\ 30^m$ in the morning. The later eclipse, however, would attain its greatest phase not long before sunset, when the disc would be sharply defined and it would be a simple matter to estimate the magnitude. There is also the inherent probability that Hipparchus would collect two observations of an eclipse in his own time more easily than two observations of an eclipse 180 years earlier. I doubt, also, whether an astronomer of −309 would be likely to think of estimating, much less of recording, the magnitude of a solar eclipse. The earliest Greek observation of a lunar eclipse, cited by Ptolemy from Hipparchus, is that of a partial eclipse at Alexandria −200 September 22,‡ where the magnitude is not recorded. From that date to −140 four more lunar eclipses are cited, of which two, in −173 and −140, were partial. In both cases the magnitude is recorded. The Babylonian astronomers of the time of Hipparchus not only observed, but predicted, the magnitudes of lunar eclipses. We should, therefore, expect an attempt to define the magnitude of a solar eclipse in −128, if the Sun were low enough, or the air thick enough, to render it practicable. We should not expect it in −309.

* This Commentary has never been printed. The passage cited and its context are given in Hultsch's paper, "Hipparchos über die Grösse und Entfernung der Sonne," *Berichte der sächsischen Gesellschaft*, 52 (1900), *Phil.-hist. Classe*, 169–200.

† *Loc. cit.*

‡ Ptolemy, *Math. Syn.*, 4, 11, ed. Heiberg, 1 (1898), 344.

I have, after finding the direction of the zone of totality, assumed as the two limits between which some part of that zone must have been situated λ26°·60, φ40°·40 and λ26°·30, φ39°·97. I have also computed the magnitude of the eclipse for Alexandria, and have made the corresponding computations both for the Hellespont and for Alexandria on the less probable assumption that this eclipse is identical with that of Agathocles.

(9) *The eclipse of Phlegon.*—Phlegon, *Olympiades*, fr. 17 (*Rerum naturalium scriptores Græci minores*, ed. Keller, 1877, 1, 101): "And Phlegon also who compiled the *Olympiads* writes about the same things in his thirteenth book in the following words: 'In the fourth year of the 202nd Olympiad (32–33 A.D.) an eclipse of the Sun took place greater than any previously known, and night came on at the sixth hour of the day, so that stars actually appeared in the sky; and a great earthquake took place in Bithynia and overthrew the greater part of Nicæa.'"

Julius Africanus,* who identifies this eclipse with the darkness at the crucifixion of Christ, attributes to Phlegon the statement that the eclipse took place at full moon. Origen,† however, declares that Phlegon made no such statement, and it will be observed that it is absent from the fragment, cited above, which was extracted from Phlegon by Eusebius.

In spite of the date given by Phlegon, it is impossible to identify this eclipse with any except that of +29 November 24, which fell in the first, not in the fourth, year of the 202nd Olympiad. The identification was first made by Kepler, *Eclogæ Chronicæ* (1615), 126.‡ Since the zone of totality must have passed through or near Bithynia, it seems reasonable to presume that it was the author's intention to place both the totality and the earthquake at Nicæa, and I have accordingly assumed that the eclipse was total at Nicæa (λ29°·60, φ40°·42). But I consider the presumption made in the case of this eclipse weaker than any of the other presumptions used in this paper. The total phase would be at the fifth hour rather than the sixth, but, as has been observed, the eclipses of Archilochus and Plutarch are also transferred to midday.

(10) *The eclipse of Plutarch.*—Plutarch, *De Facie in Orbe Lunæ*, 931 D, *Moralia*, ed. Bernardakis, 5 (1893), 434. Plutarch, *Select Essays*, 2 (1918), tr. by Prickard, 282: "Grant me that no one of the phenomena relating to the sun is so like another as an eclipse to a sunset, remembering that recent concurrence of sun and moon, which, beginning just after noon, showed us plainly many stars in all parts of the heavens, and produced a chill in the temperature like that of twilight. If you have forgotten it . . ."

The beginning of Plutarch's dialogue *De Facie* is lost, and in the part preserved to us there is no direct indication of the sup-

* Migne, *Patrologia Græca*, 10 (1857), 89.
† *In Matthæum*, Migne, *Pat. Gr.*, 13 (1862), 1782.
‡ *Opera*, ed. Frisch, 4 (1863), 435.

posed date or place of the dialogue. But something can be learned from the names of the characters. Lamprias, Plutarch's brother, probably elder brother, presided. Like Plutarch, he resided at Chæroneia, nineteen miles from Delphi, and he appears in numerous dialogues, in one of which he is, as here, the narrator. The scene of these is always laid in Greece, Ædepsus, Athens, Chæroneia, Delphi, and Eleusis having their turns. Another character, Theon, is an intimate friend of Plutarch's family, and may therefore be supposed to have lived at Chæroneia. He figures in seven other dialogues, which would all appear to belong either to Delphi or to Chæroneia, with the exception of one which the author may have intended to place at Rome.* Sextius Sylla, the Carthaginian, appears in four other dialogues, at two of which he is the host. In each case the scene is laid at Rome. Lucius, an Etruscan Pythagorean, appears in two other dialogues, as a guest of Sylla at Rome. Apollonides the geometrician, Aristoteles the Peripatetic, Pharnaces the Stoic, and the most famous character of all, Menelaus the astronomer of Alexandria, do not appear in any other dialogue. It seems easier to suppose that the Carthaginian and his Tuscan friend had wandered to Greece than that the Greeks had wandered to Rome. It will be observed that none of Plutarch's Roman friends takes part in the dialogue, and that all except the Carthaginian and the Tuscan have Greek or oriental names. The miscellaneous company would be likely to meet Lamprias and his friend nowhere so easily as at Delphi, visited as it is in other dialogues by travelling men of culture on their way between many different parts of the Roman empire.

The reference to the eclipse is made by Lucius, who is explaining to his audience the points made in a recent discussion at which he and Lamprias had been present, but none of the rest of the company. The leading part in that discussion had been taken by a person who was not present on this occasion, and who is described by more than one member of the company by the title "our comrade." Dr. Hartman† has no doubt that the anonymous comrade was intended to be none other than Plutarch himself. I do not hesitate to accept this opinion. It is difficult to see how the anonymity can otherwise be explained. It is, therefore, highly probable, if not certain, that Plutarch meant the reference to the eclipse to rest on his own authority. The description of the eclipse implies that the company might be expected to remember it, but the speaker seems to assume that some would not. εἰ δὲ μή, translated above, "If you have forgotten it," is of course simply "But, if not." It does not, as Mr. Prickard's translation suggests, imply that those who did not remember the eclipse had forgotten it. It is quite consistent with a supposition that, being at some other part of the world, they had not seen it, or had not seen it as a total eclipse. On the other hand, Lucius would hardly have used the second person plural if he and his Carthaginian friend

* Mestrius Florus is the host, but he may have had a house at Chæroneia.

† *De Plutarcho Scriptore et Philosopho* (1916), 557.

had been the only spectators of the eclipse, and it seems reasonable to suppose that Plutarch meant to imply that the eclipse had been visible to those members of the company who had been at Delphi or Chæroneia at the time. How many of these there were we have no means of saying. The description of the eclipse reads like that of an eye-witness, and I have thought it a reasonable presumption that the eclipse was total either at Delphi or at Chæroneia. "Stars in all parts of the heavens" are not seen except at a total eclipse. Struyck, *Inleiding tot de allgemeene Geographie* (1740), 101,* identified the eclipse with that of 71 March 20, and it is not easy to substitute another identification. The date falls rather earlier than those of most of Plutarch's dialogues. It is true that this identification makes the greatest phase fall about 11 a.m., instead of making the eclipse begin immediately after noon. But, as has been seen, the error in the time is in this respect similar to that made in the cases of the eclipses of Archilochus and Phlegon.

(11) *The eclipse of Theon.*—Theon, *εἰς τὸ τοῦ Πτολεμαίου βιβλίον ε̄ ὑπόμνημα* (1538), 332: ". . . the time reckoned by civil days † and equinoctial hours of the exact ecliptic conjunction which we have discussed, and which took place according to the Egyptian calendar in the 1112th year from the reign of Nabonassar $2\frac{5}{6}$ equinoctial hours after midday on the 24th of Thoth, and according to the Alexandrine calendar reckoned by simple civil days † in the 1112th year of the same reign $2\frac{5}{6}$ equal or equinoctial hours after midday on the 22nd of Payni. . . . And moreover we observed with the greatest certainty the time of the beginning of contact, reckoned by civil and apparent time as $2\frac{5}{6}$ equinoctial hours after midday, and the time of the middle of the eclipse as $3\frac{1}{5}$ hours, and the time of complete restoration as $4\frac{1}{2}$ hours approximately after the said midday of the 22nd of Payni." The date defined by Theon is clearly 364 June 16. This is the only ancient eclipse of the Sun for which an astronomically observed time is recorded. The place, Alexandria, is equally certain. It will be assumed that Theon's times are correct.

The method used in this paper is substantially the same as that used in my paper on "The Eclipse of Hipparchus," *M.N.*, 69 (1909), 204–10. I adopt elements of the motion of Sun and Moon supposed to be approximately correct, and compute the belt of totality or annularity for each eclipse except that of Theon. I make similar computations with two sets of elements in which the secular accelerations of the Moon and Sun respectively have been increased by 1″ per century. For the eclipse of Hipparchus I similarly compute three values for the magnitude at Alexandria, and for the eclipse of Theon three values for the times of beginning, greatest phase, and end of the eclipse at Alexandria. From the belts of totality I compute equations showing the limits between

* Cited by Ginzel, *Spezieller Kanon*, 203.

† *νυχθήμερον* is properly "a night and a day," *i.e.* a civil or astronomical day, as distinct from a light-day reckoned from sunrise to sunset.

which the two accelerations must lie in order to satisfy the presumptions made in regard to each eclipse. For the magnitude of the eclipse of Hipparchus at Alexandria and for the times of the different phases of the eclipse of Theon, there is one equation for each observation instead of two limiting equations. Graphs representing the different equations are then plotted out on a sheet of paper, and it is thus possible to see to what extent the presumptions are consistent and what values of the two accelerations will satisfy the presumptions which are found to be consistent.

The initial elements adopted for the Sun are those given in Newcomb's Tables of the Sun, reduced to 1800 January 0·0, with the single exception that the coefficient of T^2 in the longitude is taken as $+2''\cdot21$ instead of $+1''\cdot089$. The value adopted corresponds to a sidereal secular acceleration of $+1''\cdot10$.

For the Moon I have assumed a sidereal secular acceleration of $+10''\cdot10$ at epoch 1800 January 0·0. This is equivalent to a correction of $+4''\cdot09$ to Dr. Brown's value. Introducing this as the value for z in Table VI. of my paper on "The Longitude of the Moon," *M.N.*, 80 (1920), 303, I get by interpolation for x and y $-1''\cdot715$ and $+2''\cdot695$ respectively. These are the corrections to Dr. Brown's mean longitude and centennial motion respectively for 1810·0. Reduced to 1800·0 they become $-1''\cdot98$ and $+1''\cdot88$ respectively. Substituting $-1''\cdot98+1''\cdot88T+4''\cdot09T^2$ for the correction to Dr. Brown's mean longitude, reckoned from epoch 1800·0 as given on p. 304 of the paper cited, I obtain a correction of

$$+0''\cdot31-0''\cdot08T-0''\cdot43T^2$$

to the mean longitude of the Moon deduced in that paper. In the final mean longitude deduced on p. 305 of that paper, $25''\cdot22$ is a misprint for $25''\cdot52$. Correcting this misprint and applying the correction just found, I obtain for the preliminary value of the Moon's mean longitude

$$L=335^\circ\ 43'\ 25''\cdot83+1\bar{7}325\ 64393''\cdot68T+11''\cdot21T^2+0''\cdot0068T^3,$$

where T is the interval in Julian centuries since 1800 January 0·0.

The elements of the motion of the Moon's perigee and node are taken from Dr. Brown without change, and those of the Sun's motion from Newcomb with the one change specified above. I thus get:—

$$\begin{aligned}
g &= 110^\circ\ 19'\ 38''\cdot02+17179\ 15796''\cdot96T+48''\cdot240T^2+0''\cdot0518T^3\\
F &= 302\ 26\ 57\cdot97+17395\ 27319\cdot85T+\ 3\cdot750T^2-0\cdot0012T^3\\
L' &= 279\ 54\ 49\cdot33+\ 1296\ 02765.95T+\ 2\cdot210T^2\\
g' &= \ 0\ 24\ 41\cdot53+\ 1295\ 96580\cdot14T+\ 0\cdot616T^2-0\cdot0120T^3
\end{aligned}$$

where g and g' are the mean anomalies of the Moon and Sun respectively, F is the Moon's argument of latitude, and L′ is the mean longitude of the Sun.

The mean obliquity of the ecliptic is taken from my paper on "The Secular Acceleration of the Sun as determined from Hipparchus' Equinox Observations," *M.N.*, **78** (1918), 411, and is expressed by the formula

$$\epsilon = 23^\circ\ 27'31''{\cdot}85 - 47''{\cdot}090\mathrm{T} - 0''{\cdot}009\mathrm{T}^2 + 0''{\cdot}0018\mathrm{T}^3,$$

where T is the number of solar centuries elapsed since 1850·0.

I call these elements Set A.

A second set of elements was, as explained above, based on the assumption that the preliminary value of the Moon's secular acceleration was to be increased by $+1''$ per century. In order to make this correction harmonise with the modern observations, a further correction of $-0''{\cdot}77 + 0''{\cdot}41\mathrm{T}$ has to be applied to the Moon's longitude at epoch 1810·0.* The total correction reduced to epoch 1800·0 becomes

$$-0''{\cdot}80 + 0''{\cdot}21\mathrm{T} + 1''{\cdot}00\mathrm{T}^2.$$

This correction has to be applied to the values given above for L, g, and F. The other elements remain unchanged.

The elements as thus corrected are called Set B.

The third set of elements is obtained by applying a correction of $+1''$ per century to the secular acceleration of the Sun as assumed in Set A. I have no such elaborate discussion of modern observations of the Sun as would enable me to postulate the resulting corrections to the epoch and modern centennial motion of the Sun. The total correction to be applied to the Sun's longitude at epoch 1800·0 is therefore

$$+1''{\cdot}00\mathrm{T}^2.$$

This correction has to be applied to the values assumed in Set A for L′ and g'. The other elements of Set A remain unchanged.

The elements thus corrected are called Set C.

The computation with these three sets of amended elements was made with the aid of syzygy tables constructed by Herr Schoch, which to a large extent supersede Oppolzer's *Syzygien-Tafeln*.† Herr Schoch's tables have never been published, but he has been kind enough to supply me with a copy. In these tables the secular terms and secular alterations in the coefficients of periodic terms are more accurately represented than in Oppolzer's tables, and Newcomb's value for the eccentricity of the Earth's orbit with its secular changes takes the place of Leverrier's.

Herr Schoch has also diminished Oppolzer's value for the ratio which the radius of the Moon's figure bears to the Earth's equatorial radius. Oppolzer, following Hansen, had taken this as 0·27296. Herr Schoch has adopted 0·27252, which he believes to be correct. For occultations the *American Ephemeris* now uses

* See Table VI. in my paper in *M.N.*, **80**, 303.

† *Publication der astronomischen Gesellschaft*, **16** (1881).

Hansen's value for this quantity when the effect of irradiation is included, and the same value as Herr Schoch when the effect of irradiation is ignored. The value now used for eclipses in the ephemerides is 0·27227. The effect of using too large a value for this quantity is to widen belts of totality and to narrow belts of annularity. In view of the omission of small terms and the possibility of errors in some of the other terms, it is perhaps safer to allow a little margin and to adopt slightly broader belts of totality than the most modern figures would give. This is particularly the case in the present paper, where the eclipses are with one exception total. I have, therefore, thought it sufficient to accept Herr Schoch's correction of this term. As the term in question forms a part of the expression for the radius of the penumbra on the fundamental plane, expressed in terms of the Earth's equatorial radius, Oppolzer's u_a', Herr Schoch's tables apply a constant correction of $-0{\cdot}00044$ to that term in addition to the corrections arising from changes in the arguments of the periodic terms. The same correction has to be applied to the radius of the umbra on the same plane, Oppolzer's u_i'. This is best done by applying the correction $-0{\cdot}00088$ to the constant in Oppolzer's formula, $u_i' = 0{\cdot}5473 - u_a'$. We thus have $u_i' = 0{\cdot}5464 - u_a'$.

The computation of the belts of totality or annularity proceeds by the formulæ given in Oppolzer's *Syzygien-Tafeln*, 20, 21, [53], and repeated in his *Canon der Finsternisse*,* IX, XXIII. A few simplifications have been introduced which do not affect the accuracy of the result within the limits attempted, and in addition to the inequalities used by Oppolzer, the long-period term $-0^{d}{\cdot}00033 \sin(212^{\circ}{\cdot}4 + 132^{\circ}{\cdot}92T)$ has been introduced into the time of each conjunction, and the long-period term $+0^{d}{\cdot}000165 \sin(211^{\circ}{\cdot}0 + 20^{\circ}{\cdot}20T)$ into the time of conjunction for the eclipses of Phlegon and Plutarch. A correction of $+0^{\circ}{\cdot}001$ has been applied to all the terrestrial latitudes determined, in order to make them agree with Dr. Brown's value for the ellipticity of the Earth's figure, viz. 1/294. The latitudes on the northern and southern limits of each belt corresponding to the presumed longitudes were found by interpolation, except in the case of the eclipse of Plutarch, where, owing to the direction of the belt, it was thought better to find the longitudes corresponding to the presumed latitudes. A comparison of the results obtained by the different sets of elements showed what proportion of the difference between the elements was permissible consistently with the original presumptions concerning the eclipses. The conclusion from each eclipse is expressed by means of two linear equations, which, when represented by graphs on a diagram, show the limits within which the two accelerations must fall.

For the eclipse of Hipparchus the magnitude at Alexandria is computed by the formulæ given in Oppolzer's *Canon der Finsternisse*,

* *Denkschriften der kaiserlichen Akademie der Wissenschaften, math.-naturw. Classe*, 52 (Vienna, 1887).

XXIV, XXV, except that for the final equation I have substituted a more accurate one:—

$$\text{Greatest phase} = 12\frac{u_a - m}{u_a - u_i},$$

where $u_a = u_a' - \eta f_a \sin\delta' - \xi f_a \cos\delta' \cos t,$

$u_i = u_i' - \eta f_i \sin\delta' - \xi f_i \cos\delta' \cos t,$

the symbols on the right-hand side of each equation having the same meaning as in Oppolzer. The effect of the alterations in the elements made in Sets B and C is determined as before, and an equation is found showing values for the two accelerations satisfying the recorded magnitude. The magnitude of the eclipse of Thucydides at Athens is determined in the same manner.

For the eclipse of Theon the time of greatest phase is found by the formulæ given in Oppolzer's *Canon*, XXIV, and the times of beginning and ending by the formulæ on p. XXVIII of the same work.

The computation of the corrections to be applied to Herr Schoch's tables to make them accord with the elements of Sets A, B, C and the computation of the belts of totality or annularity for each of the first ten eclipses were duplicated by Herr Schoch, whose assistance I have found of the greatest value throughout.* Table I. gives some of the principal stages in each computation. For the meanings of the symbols used see Oppolzer's *Canon*, VIII, IX. Table II. gives in the case of eclipses (2), (3), (4), (7), (8) the points where the southern boundary of the belt of totality crosses the meridian on which the northern extreme permitted by the historical evidence as discussed above is situated, and the point where the northern boundary of the belt of totality crosses the meridian on which the southern extreme permitted by the historical evidence is situated. Where, as in the cases of (1), (5), (9), a definite point appears to be indicated by the historical evidence, the intersection of both limits of the belt of totality with the meridian passing through that point is shown. For (6) the intersection of the southern boundary of the belt of annular eclipse with the meridian of Athens is shown. For (10) the table gives the intersection of the southern boundary of the belt of totality with the parallel of latitude of Delphi, and the intersection of the northern boundary of the belt of totality with the parallel of latitude of Chæroneia. According to the values used in 4C, the total phase in longitude 38°·50 falls after sunset.

The linear equations for the two accelerations were then determined as follows:—Let γ, δ be the given latitudes corresponding respectively to the two given longitudes α, β for an eclipse. Let γ_1, γ_2, γ_3 be the latitudes computed above, corresponding to longitude α according to the three sets of elements A, B, C, and

* Herr Schoch has not duplicated my computation of the eclipse of −309 for the Hellespont.

TABLE 1.

No.	Date.	G.M.T. of Conjunction.		L′.	Z.	ϵ.	P.	Q.	log *p*.	log ΔL.	log *q*.	*u a*.	log f_a.	μ.	γ.
		h	m												
1A	– 1062 July 31	5	29·9	116°·564	+0°·12	23°·826	178°·150	176°·13	0·7005	9·7528	8·7490	0·5391	7·6669	262°·78	+0·1612
1B	,,	5	3·4	116·547	,,	,,	178·132	176·11	,,	,,	,,	,,	,,	256·15	+0·1628
1C	,,	5	55·1	116·804	+0·13	,,	178·394	176·36	0·7006	,,	,,	,,	,,	269·00	+0·1399
2A	– 762 June 15	8	21·1	74·761	−2·09	23·789	176·773	174·93	0·6988	9·7554	8·7509	0·5353	7·6627	308·07	+0·2800
2B	,,	8	0·0	74·747	,,	,,	176·759	174·92	,,	,,	,,	,,	,,	302·81	+0·2812
2C	,,	8	40·4	74·949	2·07	,,	176·963	175·12	0·6989	9·7553	8·7508	,,	,,	312·84	+0·2636
3A	– 647 Apr. 6	8	34·2	9·080	+0·71	23·774	171·798	172·44	0·6909	9·7638	8·7594	0·5319	7·6649	309·61	+0·6968
3B	,,	8	14·9	9·067	,,	,,	171·784	172·42	,,	,,	,,	,,	,,	304·78	+0·6979
3C	,,	8	51·9	9·255	+0·70	,,	171·972	172·60	,,	,,	8·7595	,,	,,	313·99	+0·6822
4A	– 584 May 28	14	34·4	59·713	−2·33	23·770	3·682	4·20	0·6906	9·7640	8·7590	0·5303	7·6623	40·13	+0·3134
4B	,,	14	15·9	59·700	,,	,,	3·669	4·19	,,	,,	,,	,,	,,	35·52	+0·3123
4C	,,	14	50·5	59·876	,,	,,	3·846	4·36	,,	,,	,,	,,	,,	44·12	+0·3273
5A	– 462 Apr. 30	12	1·6	33·404	−1·38	23·752	175·835	174·21	0·6971	9·7575	8·7529	0·5345	7·6632	2·70	+0·3598
5B	,,	11	45·3	33·394	,,	,,	175·823	174·19	,,	,,	,,	,,	,,	358·64	+0·3608
5C	,,	12	16·7	33·553	−1·41	,,	175·983	174·35	0·6972	9·7574	8·7528	0·5346	,,	6·48	+0·3471
6A	– 430 Aug. 3	14	51·4	124·629	+0·53	,,	170·757	173·14	0·7232	9·7277	8·7263	0·5531	7·6665	44·65	+0·8450
6B	,,	14	35·2	124·618	,,	,,	170·745	173·13	,,	,,	,,	,,	,,	40·62	+0·8463

TABLE I.—*continued.*

No.	Date.	G.M.T. of Conjunction. h	m	L′.	Z.	ε.	P.	Q.	log *p*.	log ΔL.	log *q*.	*w′a*.	log *fa*.	*μ*.	*γ*.
6C	− 430 Aug. 3	15	7·6	124°·775	+0°·53	23°·752	170°·903	173°·29	0·7232	9·7278	8·7263	0·5530	7·6665	48°·68	+0·8320
7A	− 309 Aug. 15	8	6·7	136·698	,,	23·733	3·896	2·69	0·6928	9·7612	8·7574	0·5350	7·6679	300·30	+0·3333
7B	,,	7	52·3	136·689	,,	,,	3·887	2·68	,,	,,	,,	,,	,,	296·70	+0·3325
7C	,,	8	20·1	136·831	,,	,,	4·029	2·80	,,	9·7613	,,	0·5349	,,	303·62	+0·3446
8A	− 128 Nov. 20	13	7·2	236·193	−2·76	23·711	9·545	11·53	0·7001	9·7518	8·7498	0·5454	7·6775	17·44	+0·8271
8B	,,	12	55·3	236·184	,,	,,	9·536	11·52	,,	,,	,,	,,	,,	14·45	+0·8264
8C	,,	13	19·2	236·308	,,	,,	9·659	11·64	0·7000	9·7519	8·7499	,,	,,	20·40	+0·8368
9A	+ 29 Nov. 24	9	24·5	240·975	−2·50	23·693	171·338	169·56	0·6969	9·7550	8·7526	0·5437	,,	325·53	+0·7457
9B	,,	9	14·4	240·967	,,	,,	171·330	169·55	,,	,,	,,	,,	,,	323·02	+0·7464
9C	,,	9	34·6	241·072	−2·49	,,	171·436	169·66	0·6970	9·7549	,,	,,	,,	328·02	+0·7375
10A	+ 71 Mar. 20	9	30·5	357·578	+2·08	23·690	7·256	9·69	0·7138	9·7380	8·7354	0·5479	7·6678	318·81	+0·6503
10B	,,	9	21·0	357·571	,,	,,	7·249	9·68	,,	,,	,,	,,	,,	316·44	+0·6497
10C	,,	9	40·0	357·666	,,	,,	7·346	9·78	0·7137	9·7381	,,	0·5478	,,	321·17	+0·6582
11A	+ 364 June 16	12	54·7	85·110	−0·83	23·651	6·437	8·28	0·6991	9·7551	8·7505	0·5353	7·6625	13·08	+0·5581
11B	,,	12	48·1	85·106	,,	,,	6·432	8·27	,,	,,	,,	,,	,,	11·42	+0·5576
11C	,,	1	0·9	85·169	−0·84	,,	6·495	8·34	0·6990	,,	8·7506	,,	,,	14·62	+0·5630

TABLE II.

	Longitude.	Latitude.	Longitude.	Latitude.		Longitude.	Latitude.	Longitude.	Latitude.
	°	°	°	°		°	°	°	°
1A	44·50	31·30	44·50	32·43	6A	23·72	47·85		
1B	,,	29·35	,,	30·45	6B	,,	52·14		
1C	,,	31·69	,,	32·85	6C	,,	42·07		
2A	42·19	37·61	45·15	39·89	7A	15·50	36·99	15·00	38·66
2B	,,	36·72	,,	39·20	7B	,,	36·32	,,	37·95
2C	,,	37·23	,,	39·28	7C	,,	38·21	,,	39·91
3A	24·56	39·17	25·25	43·08	8A	26·60	40·27	26·30	40·55*
3B	,,	36·73	,,	40·57	8B	,,	39·21	,,	39·49*
3C	,,	40·21	,,	44·03	8C	,,	42·38	,,	42·57*
4A	38·50	33·48	27·50	39·69	9A	29·60	39·41	29·60	40·44
4B	,,	35·04	,,	41·10	9B	,,	41·51	,,	42·49
4C	,,	[32·80]	,,	39·27	9C	,,	36·50	,,	37·56
5A	23·32	39·56	23·32	41·40	10A	22·78	38·48	22·38	38·50
5B	,,	39·33	,,	41·20	10B	25·22	38·48	24·82	38·50
5C	,,	38·92	,,	40·72	10C	19·56	38·48	19·14	38·50

let δ_1, δ_2, δ_3 be the computed latitudes corresponding similarly to longitude β. Put L = secular acceleration of the Moon, and L′ = secular acceleration of the Sun. Then the limiting values of L and L′ are given by the two equations—

$$L' + \frac{\gamma_1 - \gamma_2}{\gamma_1 - \gamma_3} L'' = \frac{\gamma_1 - \gamma + 10'' \cdot 10(\gamma_1 - \gamma_2)}{\gamma_1 - \gamma_3} + 1'' \cdot 10,$$

$$L' + \frac{\delta_1 - \delta_2}{\delta_1 - \delta_3} L = \frac{\delta_1 - \delta + 10'' \cdot 10(\delta_1 - \delta_2)}{\delta_1 - \delta_3} + 1'' \cdot 10.$$

Where the computation has been made for one meridian only, δ will be $= \gamma$. Where, as in (10), the computation has been made for given parallels of latitude instead of given meridians, α and β must represent the given latitudes, and γ, γ_1, γ_2, γ_3, δ, δ_1, δ_2, δ_3 must be taken to be longitudes.

The resulting equations are :—

[TABLE

* If the eclipse of Hipparchus be identified with that of Agathocles, these figures should be 40°·12, 39°·71, 41°·07. These figures should be compared with the figures in the two left-hand columns for 7A, 7B, 7C.

TABLE III.

(1)	Upper limit	$L' - 5{\cdot}00L = -46''{\cdot}27$	(6)		$L' - 0{\cdot}74L = -4''{\cdot}66$
	Lower ,,	$L' - 4{\cdot}71L = -46{\cdot}26$	*(7)	Upper limit	$L' - 0{\cdot}55L = -3{\cdot}27$
(2)	Upper ,,	$L' + 1{\cdot}13L = +21{\cdot}87$		Lower ,,	$L' - 0{\cdot}57L = -6{\cdot}32$
	Lower ,,	$L' + 2{\cdot}34L = +25{\cdot}28$	(8)	Upper ,,	$L' - 0{\cdot}50L = -3{\cdot}89$
(3)	Upper ,,	$L' - 2{\cdot}35L = -21{\cdot}05$		Lower ,,	$L' - 0{\cdot}52L = -4{\cdot}44$
	Lower ,,	$L' - 2{\cdot}62L = -31{\cdot}71$	(9)	Upper ,,	$L' - 0{\cdot}71L = -6{\cdot}06$
(4)	Upper ,,	$L' - 3{\cdot}36L = -26{\cdot}44$		Lower ,,	$L' - 0{\cdot}72L = -6{\cdot}52$
	Lower ,,	$L' - 2{\cdot}29L = -33{\cdot}09$	(10)	Upper ,,	$L' - 0{\cdot}76L = -6{\cdot}49$
(5)	Upper ,,	$L' + 0{\cdot}29L = +\ 8{\cdot}56$		Lower ,,	$L' - 0{\cdot}75L = -6{\cdot}65$
	Lower ,,	$L' + 0{\cdot}36L = +\ 6{\cdot}68$			

The words "upper" and "lower" in this table refer to the position of the graphs on the accompanying diagram.

The magnitudes of the eclipse of Thucydides at Athens and of Hipparchus at Alexandria are found to be as follows. They are expressed in digits, *i.e.* in twelfths of the diameter.

	A.	B.	C.
(6)	10·53	10·10	11·21
†(8)	9·43	9·75	8·92

As the recorded magnitude of the latter is 9·6, the resulting equation is

$$L' - 0{\cdot}62L = -5''{\cdot}48.\ddagger$$

For the eclipse of Theon the hour angles of the different phases are as follows:—

	A.	B.	C.	Observed.
Beginning	52°30	50°32	54°26	42°50
Greatest phase	65·66	63·62	67·53	57·00
End	77·86	75·78	79·65	67·50

The resulting equations are:—

(11)	Beginning	$L' - 1{\cdot}01L = -14''{\cdot}10$
	Greatest phase	$L' - 1{\cdot}09L = -15{\cdot}08$
	End	$L' - 1{\cdot}16L = -16{\cdot}42$

The graphs representing the equations show at a glance how

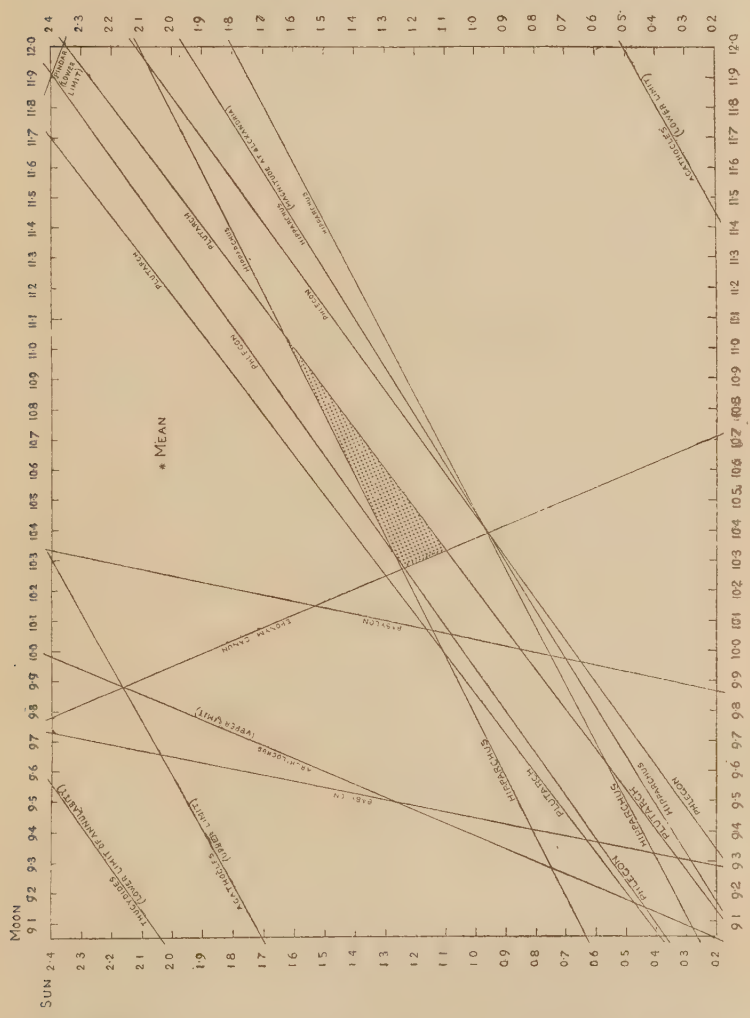
* If the eclipses of Agathocles and Hipparchus be identified, the equations for (7) and (8) combined are:—

Upper limit	$L' - 0{\cdot}55L = -3''{\cdot}27$
Lower ,,	$L' - 0{\cdot}43L = -3''{\cdot}42$

† These figures should be 9·14, 9·29, 8·82 if the eclipse be identified with that of Agathocles.

‡ $L' - 0{\cdot}44L = -4''{\cdot}75$ if the eclipse be identified with that of Agathocles.

far the initial presumptions made in this paper are consistent. All values represented on the diagram fall between the two limiting equations for the eclipse of Thales. On the other hand, the equations resulting from the eclipse of Theon fall entirely outside


MOON
SUN
* MEAN
PTOLEMY
HIPPARCHUS
PLUTARCH
PHILOCOMUS
THEON ALEX.
THEON SMYRN.
ARISTARCHUS
HIPPARCHUS
PTOLEMY
PHILOCOMUS
PLUTARCH

phases of the eclipse. He would appear to have observed the intervals with some approach to precision, but to have made a large initial error in the time.

It would appear that the presumption that the eclipse of Pindar was total at Thebes is not justified. The eclipse of Thucydides cannot have been annular at Athens, but the record seems to imply that it was only partial. Nos. (3), (7), (8), (9), (10) are found to be perfectly consistent, but it is impossible to reconcile with them both the eclipse of Babylon and the eponym canon eclipse, if we assume, as has been done in this paper, that the former was total in Babylon and the latter somewhere in Assyria. Now it seems easier to suppose that the phenomenon recorded in the Babylonian chronicle was something other than an eclipse, or, if an eclipse, was total in southern Babylonia and not at Babylon itself, than to suppose that the eponym canon eclipse was total in no part of Assyria. That chronicle does not record events outside Assyria except where the Assyrians were fighting, and it is difficult to see why an eclipse which was only known as partial should be recorded at all.

I take it, then, that the evidence of the ancient eclipses of the Sun is best satisfied by values falling within the triangle shaded on the diagram. This triangle is defined by the eponym canon eclipse and those of Hipparchus and Plutarch. It would be difficult to shake the presumptions made concerning the eclipse of Hipparchus, but those made concerning the eponym canon eclipse and the eclipse of Plutarch fall short of certainty, and, if either of these is abandoned, a larger area on the diagram becomes available. I have thought it desirable, therefore, to compare the result deduced from the solar eclipses with those deduced from four other classes of evidence and set forth in my paper in *M.N.*, 80, 581.

Using L for the lunar acceleration, L′ for the solar, and D for the difference, the following table exhibits the values derived from different classes of observations:—

	Lunar Eclipse Times.	Lunar Eclipse Magnitudes.	Occultations, etc.	Equinoxes.
L	...	...	$+10''\cdot8\pm0''\cdot70$	...
L′	...	$+1''\cdot78\pm0''\cdot45$	...	$+1''\cdot93\pm0''\cdot27$
D=L−L′	$+7''\cdot9\pm1''\cdot3$			

The weighted means derived from these four classes are $L=+10''\cdot61$, $L'=+2''\cdot04$, where L′ comes out with a weight seven times as great as that attaching to L. I have shown this mean on the diagram. It is clear that the apex of the triangle shaded in the diagram would come nearer to this mean than any point within the triangle. This would give $L=+11''\cdot0$, $L'=+1''\cdot6$. It is, perhaps, an extreme course to push both the eclipse of Plutarch and the eclipse of Hipparchus to the extreme limit of permissible values, and I therefore propose to adopt $L=+10''\cdot8$, $L'=+1''\cdot5$ as the best values attainable on the evidence before

us. The nearer to the apex we fix our values, the further we bring the eponym canon eclipse into Assyria, and the nearer we bring the eclipse of Pindar to totality at Thebes.

With these values I find that the belt of totality of the eclipse of −1062 July 31 should have passed through southern Babylonia, covering the cities of Eridu, Lagash, Larsa, Ur, and Karbel-matati, and the great astronomical centre of Erech. I find that its greatest phase at Babylon should have been at $6^h\ 44^m$ a.m. local solar time, and that its magnitude at Babylon should have been 11·54. If the phenomenon described is an eclipse, this would appear to satisfy the record sufficiently.*

If we adopt these values, the value derived from the occultations is confirmed absolutely, the lunar magnitudes are satisfied within 62 per cent., the lunar eclipse times within 108 per cent., and the equinoxes within 159 per cent. of their respective probable errors. This is a higher standard of agreement than the theory of probability would lead us to expect, and the mutual agreement of these five lines of inquiry is at once a confirmation of the value of each class of evidence as treated in the discussions in which the conclusions were drawn, and a reason for believing that the values now given cannot be in error by more than a small fraction of a second.

The substitution of +10″·8 for +10″·53 as the value of the lunar acceleration involves some slight modifications in the other constants of the Moon's longitude given in my paper in *M.N.*, 80, 305. Interpolating in Table VI. of that paper and reducing to epoch 1800·0, I obtain for the mean longitude of the Moon and the great empirical term

$$335^\circ\ 43'\ 25''{\cdot}26 + 1336^r\ 307^\circ\ 53'\ 13''{\cdot}82\mathrm{T} + 11''{\cdot}91\mathrm{T}^2 + 0''{\cdot}0068\mathrm{T}^3 + 13''{\cdot}60 \sin (104^\circ{\cdot}2 + 139^\circ\ \mathrm{T}),$$

where T is the interval in Julian centuries since 1800 January 0·0, G.M.T.

It may be well to repeat the caution that the precise values given for the two accelerations represent only the mean accelerations between the ancient observations, which fall mainly between −300 and +150, and the modern observations of precision which begin about 1658. The mean motion adopted for modern times is based on the assumption that only one empirical term of very long period is to be used, and the term actually used has a period of 258 years, so that its mean value through the range of modern observations is zero. By adopting two or more empirical terms of long period it would be possible to obtain a different mean motion for modern times and therefore a different acceleration, but in the present state of our knowledge such a mean motion would be a mere mathematical contrivance. The mean acceleration found

* The proposed values satisfy both identifications of the eclipse of Hipparchus for the Hellespont and give 9·5 as the magnitude at Alexandria if the eclipse of −128 is taken as that of Hipparchus, 9·1 if that of −309 is adopted. If the latter was total anywhere on the Hellespont, its magnitude at Alexandria cannot have exceeded 9·2.

in this paper is the actual mean acceleration of the last two thousand years. It may or may not have been subject to fluctuations.*

It may also be observed that, although the solar eclipses give no indication of the acceleration of the Sun which could not equally be explained by a negative acceleration of the Moon's node, the agreement with the ancient occultations and equinox observations which are independent of the position of the node shows that there is no room left for any addition to the theoretical acceleration of the node, and I am thus able to confirm the conclusion which I announced in *M.N.*, 78, 423, that the agreement between the observed and theoretical motions of the Moon's node has held good for the last two thousand years.

I repeat my main conclusion :—

All classes of ancient observations, including the solar eclipses discussed in this paper, are adequately satisfied by the following accelerations :—

Sidereal secular acceleration of the Moon = + 10″·8 per century.
„ „ „ „ Sun = + 1″·5 „

University Observatory, Oxford:
1920 *December* 1.

Observations of Saturn 1920 *November* 6 to *November* 20. By P. H. Hepburn, M. A. Ainslie, W. H. Steavenson, and R. L. Waterfield.

According to the elements of H. Struve, the date of passage of the earth through the ring-plane of Saturn was 1920 November, $7^d\ 5^h$ G.M.T. (about), after which date the unilluminated face of the ring would be presented to the earth.

By kind permission of the Astronomer Royal (to whom the authors desire to express their indebtedness), the following observations were made with the 28-inch refractor of the Royal Observatory, Greenwich. The power used was 450 in each case.

November 6, $17^h\ 00^m$ to $17^h\ 10^m$ (approx.). Observers P. H. H. and W. H. S. Seeing very poor, with gathering clouds which wholly obscured the planet after a few minutes' observation. The ring was conspicuous, and, so far as could be judged under the adverse conditions, the ansæ were continuous and regular.

November 7, $16^h\ 30^m$ to $18^h\ 00^m$ (approx.). Observer M. A. A. Clear sky, seeing pretty good, occasionally very good. Not the slightest trace of ring seen. The shadow of the ring on the planet was well defined; sharp and black to south, fuzzy to north, the edge on the latter side being distinctly concave. In the centre the breadth of the shadow was estimated as being about twice the

* The values here proposed satisfy the eclipse of + 1241 October 6, placing the belt of totality wholly within the well defined, but somewhat wider, zone within which the records make it clear that stars were visible. But that eclipse is satisfied by most of the values that have been proposed.

thickness of a micrometer wire, or perhaps a little less. There was a marked difference of colour between the northern and southern hemispheres, the former being bluish green, the latter yellow brown.

November 13, $16^h\ 00^m$ to $18^h\ 00^m$ (approx.). Observers M. A. A. and R. L. W. Sky clear till just before observation began, then general haze, which increased, and at times rendered Saturn almost invisible to the naked eye. Seeing poor. M. A. A. could see no trace of ring. R. L. W. at first suspected a faint linear equatorial projection extending to about one-fifth of the diameter of the planet from the limb, but later, with slightly better seeing, he saw no trace of this, and concluded that the ring was wholly invisible.

November 16, $14^h\ 30^m$ to $18^h\ 00^m$ (approx.). Observer W. H. S. Transparent sky. Seeing poor at first, but improving with increasing altitude of planet. Nothing seen of ring during first hour, though Enceladus was often glimpsed. From $15^h\ 30^m$ to $16^h\ 00^m$ ring was repeatedly glimpsed, and appeared continuous when seen. Enceladus now easy. From $16^h\ 00^m$ onwards the ring was held pretty steadily, and in better moments seen to be unequally bright. The brightest portion was estimated to correspond to a point just internal to the outer edge of ring C. From this point it faded slightly inwards, but could be traced right up to ball. Outwards from the brightest part there was a pretty sudden drop in luminosity, estimated to correspond to the inner half of ring B. Thence the brightness increased gradually to a maximum some little way inside the Cassini division, and then faded off again gradually until it was lost through faintness at a point somewhat short of the extreme outer limit of the ring system. There was no actual discontinuity at any point; nor was there any marked bulging of the brighter portions, which were of considerable length and quite unlike satellites. The ring appeared to taper at both ends to fine points. Both ansæ were equally bright. The whole ring was a most delicate object, and was certainly much more difficult to see than Enceladus. No trace of ring was seen across the ball.

November 20, $17^h\ 20^m$ to $19^h\ 03^m$. Observers P. H. H. and M. A. A. Transparency good, seeing very fair. The ring was seen at first glance, but the details required close attention. During the observation Enceladus was near eastern elongation, Mimas near western, thus affording convenient standards for estimation of the brightness of the ring. The following are from notes from P. H. H., in which M. A. A. concurs :—

$17^h\ 20^m$ to $17^h\ 30^m$. Ring clearly visible on each side; *following* ansa the brighter. Very bright condensation like a satellite on this ansa, with a long condensation near ball, brightest at end furthest from planet. Enceladus follows. On the *preceding* side ring not always seen as a continuous whole, but brightest in two places, which are not satellite-like points. Mimas precedes.

$18^h\ 00^m$. Very clear [? steady] seeing. Ring continuously held on both sides. The supposed bright condensation on the *following* ansa is Tethys, situate southwards of the outer condensation on that side, which can be seen extending from Tethys

towards the planet. On the *preceding* side the condensations are not always apparent. They seem about as bright as Mimas. [*Note.* This was afterwards found to be an underestimate.]

The rings were closely watched as dawn advanced, with a view to estimating their brightness relative to the satellites. The following comparisons were made:—

$18^h\ 22^m$. Ring still quite clear. Condensations seen on both sides (sketch).

$18^h\ 27^m$. Seeing good. Tethys clear of ansa, Mimas glimpsed. Ring as a whole produces stronger impression than Mimas. Of the *preceding* condensations the inner is the brighter.

$18^h\ 29^m$. Seeing good. Brightest parts of inner condensations just outside ball [*i.e.* not touching ball, but a little further out]. Darkest parts of ring are between the condensations. Mimas quite bright by glimpses. Continuity of rings still unbroken.

$18^h\ 34^m$. No change. Mimas well seen by averted vision. [*Note.* This was the last time Mimas was noted as being visible.]

$18^h\ 42^m$. Little change. Enceladus quite easy. Ring plain, brightest near limb.

$18^h\ 46^m$. Enceladus seen with Saturn behind occulting bar. Mimas not seen. Ring visible, condensations perceived.

$18^h\ 49^m$. Enceladus not seen. Ring visible.

$18^h\ 52^m$. Ring very faint, but perceived.

$18^h\ 54^m$. Planet behind bar. Ring visible. Tethys visible.

M. A. A. observing between this observation and the next could not be sure that he saw the ring. At the time of the next observation Saturn could not be seen by the naked eye, directed along the telescope. The following notes were made by P. H. H.:—

$19^h\ 01^m$. Ring still seen, *following* ansa full length by averted vision, *preceding* ansa near ball. Tethys visible.

$19^h\ 03^m$. Ring very doubtful. Tethys seen.

At this point observations ceased. Sunrise at $19^h\ 29^m$.

It thus appears that on this date the rings were considerably easier than either Mimas or Enceladus. The darkest part of the ansæ between the condensations could be seen when Mimas could not. On the next day P. H. H. saw the rings distinctly as fine broken lines with a $12\frac{1}{2}$-inch reflector. On that occasion the *preceding* ansa was the more easily seen.

As seen with the 28-inch refractor on November 20, the ansæ did not appear as lines, but as very elongated areas tapering towards their extremities, with fairly well-defined outlines. The condensations were but little brighter than the rest of the ring, and bulged little, if at all, as a result of irradiation.

With regard to the relative brightness of the ansæ, P. H. H. suggests that the relative ease of seeing the two ansæ is a subjective matter, depending in some way on the direction in which the image lies on the retina. W. H. S., on the other hand, considers that the effect may be objective, since he found both ansæ equal on November 16, but the *preceding* one quite distinctly the brighter on November 23, when observing with R. L. W. under similar conditions.

The Galactic Distribution of the Large Spiral Nebulæ.

By J. H. Reynolds. (Plate 4.)

It seems reasonable to assume that size rather than brightness should be taken as the criterion of distance in the case of objects of the same class having a sensible angular area. We should therefore regard the larger spiral nebulæ as being on the whole nearer to the galactic system than those subtending a small angular area. It is accordingly of some importance to examine the galactic distribution of the larger spirals, and the following table gives the approximate galactic latitude and longitude of known spirals whose apparent major axis equals or exceeds 10′. The table includes the so-called "spindle" nebulæ, which are now generally regarded as spirals, the planes of which lie near the line of sight. The maximum and minimum diameters are given for each nebula, and the nebulæ are tabulated in order of apparent size. Most of the dimensions are taken from *L.O.*, **13**, and *Helwân Bulletin*, No. 9.

Spiral Nebulæ of 10′ Diameter and Upwards.

Southern Galactic Hemisphere.

N.G.C.	α.	δ.	λ.	β.	Dimensions.
	h m	° ′	°	°	′ × ′
224	0 37	+40 43	87	−22	130 × 40
598	1 28	+30 9	100	−31	80 × 60
55	0 8	−40	293	−73	25 × 3
253	0 42	−25 50	55	−88	22 × 4
247	0 42	−21 18	76	−80	18 × 5
300	0 48	−39	265	−76	14 × 8
891	2 16	+41 53	107	−17	12 × 1

Northern Galactic Hemisphere.

2403	7 23	+65 49	118	+29	16 × 10
3031	9 47	+69 32	108	+39	16 × 10
4258	12 14	+47 52	102	+66	16 × 6
5457	13 59	+54 50	68	+60	16 × 16
4565	12 29	+26 32	185	+78	15 × 1
4244	12 12	+38 22	118	+74	13 × 1
5194	13 25	+47 43	70	+66	12 × 6
2903	9 26	+21 57	175	+42	12 × 5
3628	11 15	+14 8	195	+64	12 × 1·5
4631	12 37	+33 6	110	+82	12 × 1
5907	15 12	+56 41	58	+51	11 × 0·6
2683	8 46	+33 48	150	+35	10 × 1
3109	9 56	−26	228	+24	10 × 2
4236	12 12	+70 2	93	+47	10 × 2
4517	12 28	+ 0 38	263	+61	10 × 1
5236	13 31	−29 31	280	+31	10 × 8

As the galactic longitude of an object near the poles is apt to give a misleading impression as to its situation with regard to other objects, a diagram is given of each hemisphere, the centre of the circle being the galactic pole and the circumference representing the galactic equator.

Some interesting conclusions may be drawn from the table and the diagrams.

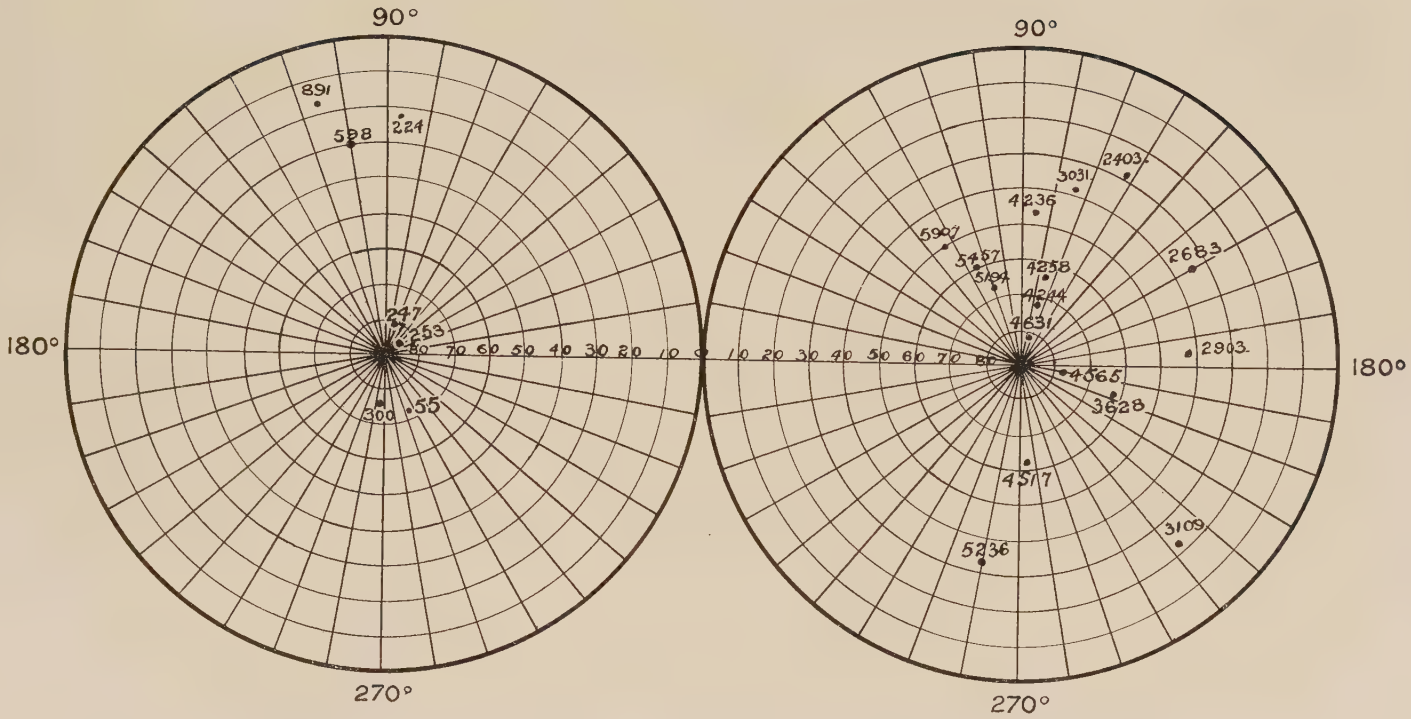
1. Five of the seven large spirals in the southern galactic hemisphere are larger than any found in the northern, and the seven are divided into two groups separated by 60° or so. There is apparently a singular absence of large spirals in the area bounded by galactic longitudes 135° and 45° (through 360°), but this is a part of the southern sky which so far has not received adequate attention with a photographic reflector of large aperture, and there may yet be found some large spirals in this region.

2. The general distribution of the spiral nebulæ has been made the subject of papers by Hinks and Hardcastle, and it has been shown that there is no such concentration of spirals near the galactic pole in the southern as exists in the northern hemisphere, the nebulæ being apparently scattered at random. This taken in conjunction with the very large size of individual spirals suggests that the nebulæ are closer to the galactic plane and to us in the southern hemisphere than in the northern. Undoubtedly spirals are much more numerous in the northern than in the southern galactic hemisphere.

3. All stages of nebular condensation are met with, the two largest (the Andromeda Nebula and M 33) being representatives of extreme types. The former consists almost entirely of amorphous nebulosity, while the latter is a collection of nebulous condensations which extend (according to Max Wolf) far beyond the apparent limits of the nebula. Some hundreds of small nebulæ are situated along the extensions of the spiral arms, and if these are actually connected with the nebula, its area is by far the largest of the spirals. From their size these two nebulæ are probably nearer to the galactic plane than any of the others, and it is curious that both are situated near the Milky Way itself. The number of stars in the field of the Andromeda Nebula is greater than with any other spiral, and the nebula seems to lie actually on the edge of a branch of the galaxy, although it is 22° distant from the galactic equator.

4. The great proportion of large spirals in which the planes are inclined at a small angle with the line of sight is very remarkable. In the southern hemisphere, five out of the seven known examples have their planes inclined at an angle of less than 30° (which would normally be taken as the probable mean). In the northern hemisphere, five are inclined at 30° and greater, and eleven at under 30°, nine of the latter being 10° and less. This seems to suggest some physical connection between the nebulæ and the centre of the galactic plane, near which the solar system is believed to be situated.

GALACTIC DISTRIBUTION OF THE LARGE SPIRAL NEBULÆ.



South Galactic Hemisphere.

North Galactic Hemisphere.

5. The distribution shown on the diagram is in accordance with Hinks' results that, whatever the law of distribution may be, it must be regarded as a function of both longitude and latitude; in fact, longitude seems the more important of the two. The hemisphere bounded by 50° to 230° contains nineteen examples, whereas the hemisphere 230° to 50° (through 360°) contains only four, two of which are very near the south galactic pole. This diagram indicates a distribution which apparently has no real reference to the Milky Way at all. Thirteen out of the twenty-three are concentrated in a comparatively small area representing about one-fifteenth of the whole sky. These are also in the region showing a great concentration of spirals, large and small.

The nebulous region in Virgo contains only one example of spirals 10′ in diameter and upwards, and the largest in the northern galactic hemisphere are to be found in Coma, Canes Venatici, and Ursa Major.

The only galactic effect which is positive is the lack of spirals generally along an area of roughly 10° each side of the galactic equator. This has been explained by the possible presence of occulting matter in and near the extension of the galactic plane. As there is no doubt as to the presence of such matter in the Milky Way itself, this is a possible explanation.

The great number of large spirals with planes little inclined to the line of sight, however, introduces another element which appears to depend on our own position in the galactic plane with regard to them. This leads to the conclusion that the larger (and presumably nearer) spirals are connected in some way with the galactic system, in which case the absence of spirals in the Milky Way regions may be a real effect.

On the Question of Stationary Radiants.

By H. C. Plummer, F.R.S.

1. The subject of the persistent radiation of meteors from certain centres is so obscure and controversial that a trivial note may find excuse. Attempts to find a general explanation of the phenomena have not been too successful, and their failure has tended, rather unjustly, to throw doubts on the observations, or at least on the inferences drawn from them by the observers themselves. It may well be that different cases require very different treatment, and that no general explanation applying to all is to be expected. The circumstances presented by radiants lying near the ecliptic and radiants near the pole of the ecliptic are widely different. In the latter case the orbits intersect the Earth's orbit at an apse, and in the ordinary notation $i = 45°$, $\omega = 0$. All that is required to give rise to a stationary radiant is a distribution of the node in longitude. This is a definite problem, of which a solution may perhaps be found on the lines suggested by Professor

Turner.* In the present note it is ecliptic radiants alone that are considered.

2. It will be assumed that the orbit of every meteor is parabolic. This may not be true, and indeed it may be supposed at the same time that the orbits are closed and periodic. The inconsistency is only one of form. All the evidence suggests that the orbits are large and therefore highly eccentric. In the neighbourhood of the Sun the motion in these circumstances is so nearly parabolic that any departure from the exact type is hardly sensible. The motion of the Earth will be taken as circular. In order to follow the history of a radiant as the Earth passes through a swarm, all that is necessary to know is the law which connects the parabolic orbits of the meteors successively met. As the problem has been reduced to one of two dimensions, what is

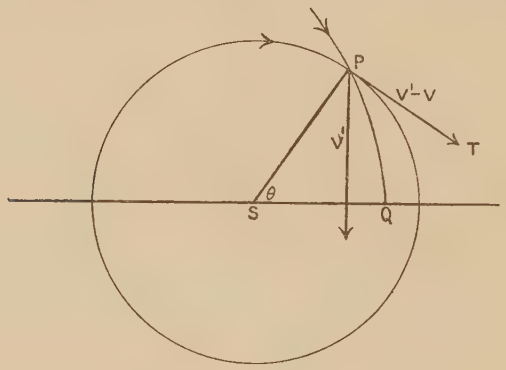


FIG. 1.

required is a knowledge of the distribution of the perihelia across a section of the stream. It seems at least interesting to consider the effect of a stream in which the orbits are coaxial. This is doubtless an artificial problem, but it seems much nearer to probable conditions than the parallel paths which have been commonly assumed.

3. Let SQ (fig. 1) be the common axis of the orbits and P the point where one of them meets the Earth's orbit, of which the centre S is the common focus of the parabolas. The velocity of the Earth being V along the tangent PT, the velocity of a meteor at P can be represented by one component V′ also along PT and an equal component V′ perpendicular to SQ. If $PSQ = \theta$,

$$2V^2 = V'^2 + V'^2 + 2V'^2 \cos\theta = 4V'^2 \cos^2 \tfrac{1}{2}\theta,$$

or $V = \sqrt{2}V' \cos \frac{1}{2}\theta$. Also, if $SP = 1$ and the perihelion distance $SQ = q$, $q = \cos^2 \frac{1}{2}\theta = V^2/2V'^2$. The components of relative velocity at P are now $V' - V$ along PT and V′ perpendicular to SQ, or

$$R \sin\phi = -(V' - V) \sin\theta, \qquad R \cos\phi = V' + (V' - V) \cos\theta,$$

* *M.N.*, 59, 140.

parallel and perpendicular to QS. Here ϕ is measured from the perpendicular to SQ($\theta = 90°$) in the direction of the Earth's motion or increasing longitude. The position of the radiant is thus given by

$$\tan\phi = \frac{(V - V')\sin\theta}{V'(1+\cos\theta) - V\cos\theta}$$

$$= \frac{(\sqrt{2}\cos\tfrac{1}{2}\theta - 1)\sin\theta}{1+\cos\theta - \sqrt{2}\cos\tfrac{1}{2}\theta\cos\theta}$$

$$= \frac{2\sin\tfrac{1}{2}\theta(\sqrt{2}\cos\tfrac{1}{2}\theta - 1)}{2\cos\tfrac{1}{2}\theta - \sqrt{2}\cos\theta}$$

$$= -\frac{\sin\theta - \sqrt{2}\sin\tfrac{1}{2}\theta}{\cos\theta - \sqrt{2}\cos\tfrac{1}{2}\theta}.$$

Hence

$$\sin(\phi+\theta) = \sqrt{2}\sin(\phi+\tfrac{1}{2}\theta),$$

and therefore

$$\tan(\phi+\tfrac{3}{4}\theta) + \tan\tfrac{1}{4}\theta = \sqrt{2}\{\tan(\phi+\tfrac{3}{4}\theta) - \tan\tfrac{1}{4}\theta\} \quad . \quad (1)$$

$$\tan(\phi+\tfrac{3}{4}\theta) = \frac{\sqrt{2}+1}{\sqrt{2}-1}\tan\tfrac{1}{4}\theta \ . \ .$$

By this relation ϕ is easily calculated for any value of θ.

4. In the above the motion of the meteors has been supposed to be direct. The corresponding result for retrograde motion is obtained by reversing the motion of the Earth or the sign of V, or again by simply changing the sign of $\sqrt{2}$. Hence it is

$$\tan(\phi'+\tfrac{3}{4}\theta) = \frac{\sqrt{2}-1}{\sqrt{2}+1}\tan\tfrac{1}{4}\theta = \tan^2\tfrac{1}{8}\pi\tan\tfrac{1}{4}\theta \quad . \quad (2)$$

but the latter form has not the same interest as the equation for direct motion,

$$\tan\tfrac{1}{8}\pi\tan(\phi+\tfrac{3}{4}\theta) = \tan\tfrac{3}{8}\pi\tan\tfrac{1}{4}\theta,$$

which shows at once the important fact that $\phi = 0$ both when $\theta = 0$ and when $\theta = \tfrac{1}{2}\pi$.

The values of ϕ for direct orbits and ϕ' for retrograde orbits, corresponding to successive values of θ and the perihelion distance q, are shown in Table I. These cover one half of the Earth's orbit. Results for the other half are obtained by reversing the signs of θ and ϕ or ϕ', but as they refer to meteors coming from inside the Earth's orbit, the corresponding radiants are not likely to be observed. Both ϕ and ϕ' are reckoned from the perpendicular to the common axis of the paths, but ϕ positive means a shift of the radiant towards greater longitude, while ϕ' positive means one in the opposite direction. Thus a direct radiant is $90° - \theta - \phi$ behind the anti-solar point in longitude, but a retrograde radiant is $90° - \theta - \phi'$ in front of that point.

TABLE I.

θ.	q.	ϕ.	ϕ'.	θ.	q.	ϕ.	ϕ'.
0°	1·00	0°	0°	90°	0·50	0°	− 63°·4
10	0·99	6·8	− 7·1	100	·41	− 5·2	− 70·4
20	·97	12·0	− 14·2	110	·33	− 10·7	− 77·4
30	·93	15·0	− 21·2	120	·25	− 16·5	− 84·3
40	·88	15·8	− 28·3	130	·18	− 22·6	− 91·3
50	·82	14·8	− 35·3	140	·12	− 28·8	− 98·2
60	·75	12·4	− 42·4	150	·07	− 35·1	− 105·0
70	·67	9·0	− 49·4	160	·03	− 41·3	− 111·8
80	·59	4·8	− 56·4	170	·01	− 48·1	− 118·6
90	0·50	0	− 63·4	180	0·00	− 54·7	− 125·3

5. As would be expected from first principles, retrograde meteors related in the manner supposed give no suggestion of a stationary radiant. On the contrary, they give a displacement with remarkable uniformity at the rate of about 2° in three days. The Perseids are perhaps an example, though their steep inclination to the ecliptic prevents them from being strictly relevant. For the Orionids the longitude of the radiant is about 90°, and that of the anti-solar point towards the end of October about 30°. According to Table I., the only way of satisfying these conditions is by a retrograde orbit with $\theta = 100°$, $\phi' = -70°$, for then the radiant is $90° - \theta - \phi' = 60°$ in front of the anti-solar point. Hence coaxial paths would give a displacement of the radiant amounting to 14° in three weeks. If the radiant is stationary, as asserted, it is to be inferred that the paths are not coaxial.

6. Meteors moving in direct coaxial orbits are more promising. Such a system gives identical radiants at points three months apart, and the displacement in the interval reaches a maximum of 15°·8. This is given by

$$\sin(\phi + \theta) = \sqrt{2}\,\sin(\phi + \tfrac{1}{2}\theta)$$
$$\sqrt{2}\,\cos(\phi + \theta) = \quad \cos(\phi + \tfrac{1}{2}\theta),$$

which give

$$\sin(2\phi + 2\theta) = \sin(2\phi + \theta),$$

or $4\phi + 3\theta = \pi$. Hence

$$2 = 4\sin^2(\phi + \tfrac{1}{2}\theta) + \cos^2(\phi + \tfrac{1}{2}\theta)$$
$$= \tfrac{5}{2} - \tfrac{3}{2}\cos(2\phi + \theta) = \tfrac{5}{2} - \tfrac{3}{2}\cos\tfrac{1}{2}(\pi - \theta),$$

or $\sin\frac{1}{2}\theta = \frac{1}{3}$. Therefore $\theta = 38°{\cdot}9$, $\phi = 15°{\cdot}8$ at the turning-point of the radiant.

Now a total range of less than 16° in three months is not large, but it is inconsistent as a whole with a stationary radiant. Nevertheless the radiant certainly is stationary at the turning-point, and the fact seems to dispose effectually of some sweeping statements that stationary radiants are quite impossible. A

range of 4° in a fixed radiant seems allowable and consistent with good observations. The interval of θ between 20° and 60° keeps the variation of ϕ within this limit, and thus the duration of the stationary position for practical purposes can be estimated at six weeks. The Earth's longitude exceeds the longitude of the radiant by 16° to 56°. At perihelion the width of the stream is 0·22.

7. These considerations may have some relation to observed results. In Table II. a comparison is made with a few radiants taken from a list by Mr. Denning.* The number in the list, the position and the duration of the observed radiant, are given in the first four columns. The positions in the fifth and sixth columns are for points on the ecliptic with the same R.A., and the dates in the seventh column show the time when the Sun is in the opposite direction. Hence can be inferred the approximate limits, given in the final column, within which a radiant in the corresponding ecliptic position may be expected to appear practically stationary.

Table II.

No.	α.	δ.	Duration.	α.	δ.	Date.	Limits.
7	14°	+ 7°	Aug.–Nov.	14°	+ 6°	Oct. 9	Oct. 22–Dec. 6
33	61	+28	Aug.–Feb.	61	+21	Nov. 26	Dec. 9–Jan. 23
46	80	+23	Aug.–Feb.	80	+23	Dec. 13	Dec. 26–Feb. 9
63	119	+30	Sept.–Apr.	119	+21	Jan. 17	Jan. 30–Mar. 16
106	217	− 7	Feb.–July	217	−15	Apr. 30	May 13–June 27
137	285	−24	Apr.–Sept.	285	−23	July 6	July 19–Sept. 2

It will be seen that the radiants are observed to be active some three months before the predicted date. It is the later stage of the activity which is tolerably represented. But it should be added that there are also ecliptic radiants which not only appear at a much earlier date, but also disappear before the expected time. The one rule is that the activity is never greatly prolonged beyond the limit assigned; but this may be, at least partly, the effect of the radiant point approaching the Sun.

8. The failure to account for the whole course of a stationary radiant does not affect the more limited result. The argument is reversible, and from the fact that radiants observed between certain dates, related in a definite way to the direction, are practically stationary, it is a valid inference that the meteors then met by the Earth are moving in orbits which are nearly coaxial. But of course there is all the difference between a radiant which is stationary for six weeks and one which is stationary for six months. Hence it is interesting to examine that distribution of parabolic orbits in the plane of the ecliptic which will give rise to a fixed radiant. This is completely determined by the distribution of the perihelia.

* *A.N.*, 3531.

Let the radiant be perpendicular to the line SX, S being the Sun (fig. 2). Let P be the point where a meteor intersects the Earth's orbit. On the radius SP construct the square SPTH, and through T draw the line RT perpendicular to SX. If PT represents the Earth's velocity, V, in magnitude and direction, PH represents the parabolic velocity, $U = \sqrt{2}V$, in magnitude. Hence, if PU = PH, PU = U represents the velocity of the meteor both in magnitude and direction, for the relative velocity is PU − PT = TU, giving a radiant in the required direction. Hence PU is the tangent to the parabolic orbit, and if PU meets the circular orbit in A, SA is the axis of the parabola, being equally inclined with the radius vector SP to this tangent. If B is the middle point of the chord PA and Q is the foot of the perpendicular from

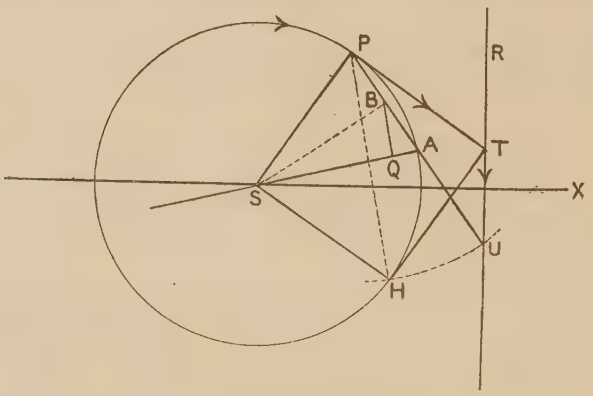


FIG. 2.

B on SA, then Q is the perihelion of the parabolic orbit, which accordingly has been found by simple geometrical construction.

9. The locus of Q is now required. Let $SQ = q$, $QSX = \phi$, $PSX = \theta$. Then $PTR = PSX = \theta$, $PUR = BSX = \frac{1}{2}(\theta + \phi)$, and therefore $V \sin\theta = U \sin\frac{1}{2}(\theta + \phi)$. Hence

$$\sin\tfrac{1}{2}(\theta + \phi) = \sin\theta/\sqrt{2} \quad . \quad . \quad . \quad (3)$$

or $\cos^2\theta = \cos(\theta + \phi)$. Also $PSB = BSA = \frac{1}{2}(\theta - \phi)$, so that

$$q = SB\cos\tfrac{1}{2}(\theta - \phi) = \cos^2\tfrac{1}{2}(\theta - \phi) \quad . \quad . \quad . \quad (4)$$

the radius SP being 1. This equation equally follows from the polar equation of the parabola.

The result of eliminating θ between (3) and (4) in such a form as

$$(9 - 8q)\sin^4\phi - 4(3 - 2q)(1 - q)\sin^2\phi + 4(2q - 1)^2(1 - q)^2 = 0 \quad (5)$$

gives the polar (q, ϕ) equation of the required locus, but it is not very useful. The rational form is not even correct, because it

fails to distinguish between directly opposite radiants. The curve must be plotted by points, for which the co-ordinates are given in Table III. This combines the two ranges, direct and retrograde,

TABLE III.

θ.	ϕ.	q.	θ.	ϕ.	q.	θ.	ϕ.	q.
0°	0°	1·00	60°	15°·6	0·86	120°	− 44°·4	0·02
10	4·2	1·00	70	13·2	0·77	130	− 64·4	0·02
20	8·0	0·99	80	8·2	0·66	140	− 86·0	0·15
30	11·4	0·97	90	0	0·50	150	− 108·6	0·40
40	14·0	0·95	100	− 11·8	0·31	160	− 132·0	0·69
50	15·6	0·91	110	− 26·8	0·14	170	− 155·8	0·91
60	15·6	0·86	120	− 44·4	0·02	180	− 180	1·00

already given in another form in Table I. For comparison the latter must be divided into two, I. (a) for θ, ϕ and I. (b) for θ, ϕ'. Then the scheme

III.	I. (a).	I. (b).	Long.
θ	$\theta+\phi$	$180°-\theta-\phi'$	$90°+l-\mathrm{L}$
ϕ	ϕ	$-180-\phi'$	$90+l-\lambda$

shows the correspondence of the notation based on a fixed radiant with that employed for a fixed position of the axis of the orbit. The last column expresses the angles in longitude, l being the longitude of the radiant, L of the Earth, and λ of the perihelion of the meteor's orbit, reckoned from the equinox.

10. The locus of perihelia is represented in fig. 3. The full curve corresponds with the upper half of the Earth's orbit, on which the meteors fall from outside; the dotted curve, which is necessary to complete the other, corresponds with the lower half of the Earth's orbit. Each is divided into two parts at the Sun, that to the left being the retrograde branch, that to the right the direct, or the perihelion locus of direct orbits. At the Sun itself there is a pair of cusps, where the tangents are given by

$$9\sin^4\phi - 12\sin^2\phi + 4 = 0,$$

the lowest terms of (5). Since $\sin^2\phi = y^2/(x^2+y^2)$, this may be written

$$(y^2 - 2x^2)^2 = 0, \quad \sin^2\phi = \tfrac{2}{3}, \quad \tan^2\phi = 2,$$

or $\phi = -54°{\cdot}7$ for the cusp on the full curve. Since $q = 0$ at this point, $\frac{1}{2}(\theta-\phi) = 90°$ and $\theta = 125°{\cdot}3$. A meteor met at this point is heading directly for the Sun on a rectilinear path which divides the retrograde from the direct orbits. The Earth arrives at the point thirty-six days before its longitude is the same as that of the radiant. Those meteors of the stationary ecliptic radiant

which just escape the Earth are destined to fall into the Sun. It would be interesting to know whether meteors have been observed at this critical point. The question is not confined to stationary radiants. The permanence of an annual shower with an ecliptic radiant in longitude 35° to the east of the anti-solar point would seem doubtful.

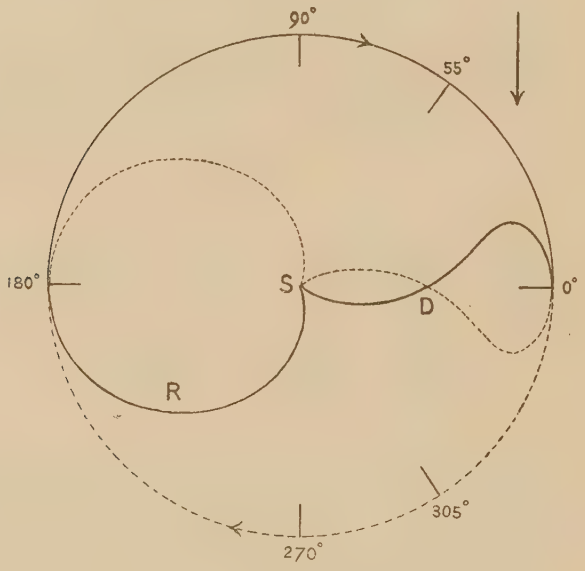


FIG. 3.

The position of tangents from the origin are given by

$$\sin\theta = \sqrt{2}\,\sin\tfrac{1}{2}(\theta+\phi)$$
$$\sqrt{2}\,\cos\theta = \cos\tfrac{1}{2}(\theta+\phi)$$

$$2 = \tfrac{5}{2} - \tfrac{3}{2}\cos(\theta+\phi) = \tfrac{5}{2} + \tfrac{3}{2}\cos 2\theta$$

or

$$-\cos 2\theta = \cos(\theta+\phi) = \tfrac{1}{3},$$

so that $\theta = 54°{\cdot}7$, $\phi = 15°{\cdot}8$. The position of the turning-point is evident in fig. 3. It is also clear that points near the point of contact of the tangent give positions of perihelion which are nearly in line with the Sun and correspond with those stationary radiants which arise from coaxial orbits. The complete locus is symmetrical about the initial line of ϕ or θ, and that part from which meteors pass out of the Earth's orbit in the same constant radiant requires no separate comment.

11. The Orionids are an interesting example. The longitude of the radiant is $l = 90°$ and of the Earth $L = 30°$. Hence in the

notation of Table III., $\theta = 90° + l - L = 150°$, $\phi = -109°$, and the longitude of perihelion (from the equinox) is $\lambda = 90° + l - \phi = 289°$. Also $q = 0{\cdot}40$. For Halley's comet $q = 0{\cdot}59$, $\Omega = 57°{\cdot}3$, $\varpi = 169°{\cdot}0$, or $\omega = 111°{\cdot}7$. But as the inclination is $i = 162°{\cdot}2$, and not 180°, the comparison of $\Omega - \omega = 305°{\cdot}6$ with $\lambda = 289°$ is not altogether justified. Moreover, the meteors observed are outlying members of the main stream, for at the ascending node the orbit of the comet is at some distance from the Earth's orbit. Yet it is interesting to see where the point $q = 0{\cdot}40$, $\phi = 251°$ falls on the curve of fig. 3, because the perihelia of the stream must be distributed approximately along an arc of the curve near this point if the corresponding radiant is to appear stationary for any appreciable time. In so far as the stream is inclined to the ecliptic the case is merely an illustration which does not altogether correspond with the conditions supposed. But there is no reason why the perihelia should not be distributed in something very like the manner here represented, or why a radiant lasting for several days or even a few weeks at a time should not appear stationary. It will be noticed that the conditions required to produce a stationary radiant on the one side of the Earth's orbit are not at all those which will have the same effect on the other side. Thus if the Orionids and the η-Aquarids are different manifestations of the same stream and the former radiant is stationary, it does not follow that the latter must be so too, but quite the contrary.

12. Stationary radiants in the ecliptic, which preserve their character for a comparatively short time at each appearance, seem to require no special explanation. They imply a particular distribution of the perihelia of the constituent orbits, but there is no very obvious reason why the necessary conditions should not be satisfied. In a given case it might even be possible to find some good reason why the arrangement should be what it is and not some other. The difficulty is associated with the long duration of certain radiants. In order to explain an ecliptic radiant which lasts for six months at a time, the perihelia must be situated along the rather peculiar curve represented in fig. 3. This is not a matter of theory beyond the simple assumption of parabolic velocity, which is supported not only by general principles but by the descriptive notes of the observers, who have expressed a belief in the reality of stationary radiants in the wider sense. It is simply an alternative mode of statement. In order that a fixed radiant in the ecliptic may be observed for several months, the perihelia must have this peculiar arrangement. It is then, among other things, necessary to explain the transition from direct to retrograde orbits. It may be compatible with the observations to suppose that the showers are intermittent rather than continuous. In that case it would be instructive to have the perihelia plotted on their characteristic locus according to the dates at which the position of the radiant was actually determined. At the least this would be an interesting form of representation of the results

of observation. For ecliptic radiants this simple method is possible, while the study of other radiants of long duration is more complicated.

13. It cannot fail to be noticed that the retrograde branch of the perihelion locus differs little from a circle on a radius of the Earth's orbit as diameter. This branch corresponds to the range $180° \pm 55°$ of θ, or about eight weeks before and after the time when the Earth's longitude is 90° less than that of the radiant. The resemblance to a circle is so close that it is natural to expect that if the circle were substituted for the true locus a nearly stationary radiant would result. Examination shows that this is so. The outer half of the circle, the semi-circle remote from the Sun, corresponds to the range $180° \pm 20°$ of θ, or about three weeks before and after the time mentioned above. During this time the variation in the position of the radiant hardly exceeds $\pm 1°$. With perihelia on the inner half of the circle the motion of the radiant becomes sensible as the Sun is approached. Naturally, when the perihelion is very close to the Sun, the radiant is very sensitive to a small change in its position. But it is remarkable, and may be significant, that a radiant stationary for some weeks can be constructed from retrograde orbits when the perihelia are distributed along a circle having the prescribed relation to the Earth's orbit. The circular locus of the perihelia imposes a condition which can equally be expressed by another property of the orbits, for it is a simple consequence that the tangent to the Earth's orbit at the point $\theta = 180°$ is also a tangent to the parabolic orbits. This common tangent is parallel to the direction of the radiant.

14. This note contains nothing in the nature of dynamical theory, and no opinion is expressed as to the real existence of persistent radiants in the widest sense. Further investigation is clearly necessary. The conception of a stream of meteors must be widened, and should perhaps be replaced by a system of orbits. The simplest natural system is in a plane, and for this reason the special study of ecliptic radiants seems to offer a promising field. Radiants within the limits $\pm 5°$ in latitude may perhaps be treated as ecliptic. For each radiant observed, with its date, there is a corresponding position of perihelion of the orbit, which can be plotted as a point within the circle which represents the Earth's orbit. When this is done the result will be to substitute for the list of radiants and dates a distribution of points within the circle. This distribution would at least present the problem in a new form, and should provide an interesting and very possibly a more profitable study.

Stationary radiants of limited duration are certainly possible, and have been illustrated by direct orbits with a common axis and by retrograde orbits with perihelia distributed along a circle. The perihelion locus of the parabolic orbits required to maintain a permanent ecliptic radiant has also been found. The necessary combination of direct and retrograde orbits to account for the

NOVA CYGNI III. 1920 SEPT. 8.
Exposed G.M.T. $9^h\ 15^m$—$10^h\ 15^m$.

complete phenomenon is an obstacle to considering the whole system as the outcome of a single origin. But the failure to find a physical explanation cannot be set against the observations. Contrary to a view often expressed, the difficulty appears to be one rather of practical improbability than of theoretical impossibility. On the other hand, if an easier explanation is found in the casual association of partial streams, rather than in a single organic system, the observer as such need have no objection. He is concerned only when his results are challenged. The problem needs to be explored with freedom in all possible ways, and one is suggested in the present note.

Note on the Spectrum of Nova Cygni III. By J. Storey. (Plate 5.)

(*Communicated by the Astronomer Royal for Scotland.*)

The spectrum of Nova Cygni III. was photographed at the Royal Observatory, Edinburgh, with a prismatic camera of 12 inches focal length, having a lens of aperture 2 inches and a 60° prism—both of quartz.

Exposures were obtained on nine days between 1920 September 4 and October 8, Barnet "Stella" plates being employed. The scale is 6·9 mm. between the lines H_β and H_μ.

It appears worth while to draw attention to the intensity of the continuous spectrum in the ultra violet.

The characteristic hydrogen bands from H_β to H_ν are well shown with a fairly faint continuous spectrum, but beyond H_ν on the more refrangible side the continuous spectrum is much more intense to about λ330, where there is a distinct diminution for a short period, the intensity increasing again and finally fading away as atmospheric absorption takes place.

The reproduction shows the spectrum on September 8, with exposure 1 hour from G.M.T. $9^h\ 15^m$–$10^h\ 15^m$, enlarged nine times from the original negative.

The comparison stars are γ Lyræ (above) and α Lyræ (below).

Behaviour of Radiations at λλ4634–41 *and at* λ4650 *in the Wolf-Rayet Stars.* By C. D. Perrine.

Dr. Lunt in his paper, *Monthly Notices, R.A.S.*, March 1920, p. 534, comments upon the fact that in Campbell's results for the Wolf-Rayet stars the bands 4634 and 4650 never appear together in the same star. This fact (as well as the duplicity of the higher radiation) was noticed by me about two years ago, and has been under investigation at intervals since. The complementary behaviour of these relations is so marked and seems so significant that a preliminary note was prepared a year ago, but as further points of interest were encountered, its publication has been delayed.

Not only do these two series of radiations appear not to be present together in the same star, but it has been found that 4634 and 4641 are present only when the lines are rather narrow. When the bands are very broad these two lines cannot be detected. Under these conditions only the radiation at 4650 (or 4649) appears to be present. Out of 22 stars, 12 with bands at 4650 showed none at 4634–41, and 10 with bands at 4634–41 showed none (with one possible exception) at 4650. The possible exception is the star A.G.C. 23073, which shows what may prove to be traces of structure at 4650. It is very faint, and consists of an absorption line with brightened edges. This detail appears to be of the same general character as that observed in some early B type stars, and is now under investigation. As all of these radiations have been identified together in the Novæ, it cannot be reasonably urged that they cannot exist together in the Wolf-Rayet stars, although but one case has been observed which can be considered as possibly contradictory.

The behaviour is so well marked, however, that it can be said that when the lines and bands are narrow 4634–41 are present, and practically no traces of emission at 4650 present; whereas when the bands are very wide 4650 is very pronounced and 4634–41 are either not present at all or cannot be detected. The star A.G.C. 23073 is probably very near the transition stage.

These characteristics may be further generalised by saying that 4650 appears to be very sensitive—the most sensitive so far detected in the ordinary photographic region, and that it is only present as strong emission where the great widths of the bands justify the belief that the star's surface is very active, due probably to high temperature. The presence in the Novæ of 4650 along with 4634 and 4641 indicates different behaviour or characteristics of these elements in the Novæ, perhaps depending upon the suddenness or violence of the outburst. The more leisurely march of the Wolf-Rayet stars probably gives time for the separate appearance of phenomena which in the Novæ are jumbled together in a small interval of time.

Wright has suggested the 4686 line as a means of classifying

the nebulæ.* The emission at 4650 appears to be more sensitive than that at 4686, and I would suggest its inclusion together with 4634–41 in any scheme for the classification of both the Wolf-Rayet stars and nebulæ. A study of the lower regions of the spectrum may lead to other radiations which should be included also in their classification.

Observatorio Nacional Argentino, Cordoba:
1920 *October* 23.

Presence of Emission at λλ4634 *and* 4641 *in the Spectra of the Wolf-Rayet Stars.* By C. D. Perrine.

Dr. Lunt, in his article on the spectrum of Nova Aquilæ No. 3 in the *Monthly Notices* for March, discusses the relation of spectra of novæ to those of the Wolf-Rayet or Class O stars. He calls attention to the presence among others of λλ4604, 4634, 4650, and 4686 in both, and concludes that the radiation at 4641 is peculiar to the novæ. This confusion as to the presence of 4641 is doubtless due chiefly to the fact that the early observers of these stars failed to separate the two lines 4634–41. An explanation for the position 4634 for the blend is not evident.

In a footnote to his article on the enhanced lines of nitrogen in *Monthly Notices* for June, Professor Fowler refers to the observation of the two lines 4634 and 4641 in the spectrum of the Wolf-Rayet star A.G.C. 22748 by Paddock,† and in several nebulæ by Wright.‡ The object of the present note is to strengthen this datum by the inclusion of other Wolf-Rayet stars in which both lines have been observed.

Spectrograms taken with the astrographic refractor and a 20° prism of 5 inches aperture placed in front, dispersion 33A per millimetre at H_γ, show that in the six stars A.G.C. 22748, 22763, 22843, 23073, C.P.D. −59° 2450 and C.P.D. −59° 2548, both radiations are present. In A.G.C. 22748 and 22843 these lines are narrow, of nearly equal intensity, and well separated. In A.G.C. 23073 the lines are less sharp, but both radiations are distinguishable. In 22763 the lines are considerably widened, yet 4634–41 is certainly double. In C.P.D. −59° 2450 and −59° 2548 the radiations are much widened, yet both 4634 and 4641 are clearly discernible. A.G.C. 22827, C.P.D. −58° 2546 and γ Argûs have the bands so widened, and the spectra of the first two are so much fainter, that they furnish no evidence on the radiations in question. These are (with ζ Puppis, to be

* *Proceedings National Academy of Sciences*, 1, 590, 1915.
† *Lick Observatory Bulletin*, 8, 78, 1914.
‡ *Publications Lick Observatory*, 13, 193, 1918.

mentioned later) all the stars of this class which have been observed here thus far.

The most prominent lines in the spectra of A.G.C. 22748 and 22843 have been measured, using the ordinary hydrogen absorption as a standard. If the resulting positions are corrected on the basis of 4686 by using Fowler's position, 4685·98, the following positions result:—

Plate of 1917 *August* 18.

A.G.C. 22748	λ 4633·9	λ 4641·9
Remeasured	4634·2	4641·8
A.G.C. 22843	4634·2	4641·3
Mean	4634·1	4641·7

These are only provisional measures and reductions of no great accuracy. It is believed, however, that the error in the above positions will not exceed 1A.

The positions of the lines in the spectra of the other stars were confirmed by direct comparison with one of these two. Thus all the stars observed whose lines are sufficiently narrow, six in number, show both radiations. As no contradictions were found, it seems justifiable to conclude that their presence is probably characteristic of the Wolf-Rayet stage *when the lines are narrow.*

Our sharpest spectra of ζ Puppis appear to show also that the radiation 4634–41 is double. It is so faint in that star, however, that it was not included with the other data.

There seems no reason to doubt that these lines are, in fact, enhanced nitrogen radiations.

Observatorio Nacional Argentino, Córdoba:
1920 *October* 23.

Second Note on the Long-period Variable W Cygni. Observations by E. F. Sawyer, 1885–1895. By H. H. Turner and Mary A. Blagg.

1. At the end of the former paper on this star (*M.N.*, **80**, 62) reference is made to the maxima and minima tabulated in *H.A.*, **55**, 236. Among these were a number by Mr. Edwin F. Sawyer, and it seemed possible that his original observations (from which the max. and min. had been deduced) might elucidate the earlier history of the star. Through the kind help of the editor of the *Astronomical Journal* communications were opened, and ultimately Mr. Sawyer sent us a valuable list of his observations (which he had laid aside and forgotten) in the years 1885–1895, "made with a field-glass magnifying four diameters and placed slightly out of focus." The covering letter is dated 1920 June 13, from 25 Lakeville Place, Jamaica Plain, Mass.

2. The following comparison stars were used:—

TABLE I.

Sawyer's Comparison Stars.

Letter	B.D.	1900·0. R.A. h m	1900·0. Dec. ° ′	R.H.P.	Sawyer's Mean in Steps.
a′	+45·3558	21 25·8	+46 6	5·34	0
a	+46·3305	21 21·6	+46 17	5·54	5
b	+44·3889	21 33·6	+44 15	6·11	11
c	+43·4002	21 36·0	+43 59	(6·70)	16
d	+45·3584	21 29·6	+45 25	6·56	20

In column 5 the magnitude for *c* (not given in R.H.P.) is taken from *H.A.*, **45**, 277.

3. A few entries will show the nature of the records. In the year 1885 we have—

	h			
June 3	10·0	W=*d*		
July 3	9·0	W3·4*d*	*b*4W	S.H.
,, 6	9·0	W4*d*	*b*4W	P.C. :
,, 8	9·0	*b*3W5*d*	W3*c*	

The remarks are, S.H. = slight haze; P.C. = partially cloudy; a colon marks an inaccurate observation and an asterisk shows moonlight.

The entries on July 3, 6, and 8 give the intervals between *b* and *d* as 7·5, 8·0, and 8·0 steps.

Collecting all such instances, the mean values were closely as shown in the last column of Table I.

If we neglect stars *a′* and *d*, which are but seldom used, the value of a step is seen (from H.P.) to be about 0·1 mag.; but the evidence of the stars *a′* and *d* would sensibly reduce this. It seems best to conduct the investigation in steps, deferring the conversion into magnitudes till the end.

4. In the previous paper it was shown that there were two coexistent fluctuations of the star, denoted by θ and ϕ, the periods being 129·6 days and 243 days. In Table II. Sawyer's observations are cut up into lengths of 16·2 days, 8 of which make up a θ cycle and 15 a ϕ cycle. The points were obtained by drawing a curve through the plotted observations, and it is considered that this summary is perhaps sufficient under the circumstances. Where observations are so scanty as to make the reading uncertain, the numbers are enclosed in brackets. It is not difficult to fill in the gaps approximately, for instance, by taking the means of numbers above and below, so as to make each line complete: and then each line was expressed in the form $R \cos(\theta - \theta_0)$. The phases are given under the heading θ_0 and the coefficients under R.

5. It is at once seen that there are two well-marked "jumps," or sudden changes of phase, after E = 14 and 24.

The mean values for the three sets (obtained from completed rows) are—

								Mean.	R.	θ_0.
158	144	121	96	94	106	136	152	126	33	− 12°
157	182	185	157	100	61	79	105	128	63	+ 65
107	145	171	179	166	126	93	90	135	50	+125

The changes of phase are thus nearly but not quite equal, and not far from 60° each, or one-third of a cycle. Within the sets the phase remains sensibly constant.

TABLE II.

Mean Value for every 16·2 *days expressed in units of* 0·1 *step.*

E.	Initial Date.	(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).	R.	θ_0.
2	9702·4	(181)	158	120	85	70	85	117	133	50	+ 7°
3	9832·0	145	140	100	80	85	105	...	...	33	− 13
4	9961·6	...	...	...	...	...	(74)	105	150	39	0
5	0091·2	158	137	110	90	89	104	145	160	39	− 23
6	0220·8	154	128	100	82	70	67	(85)	...	46	+ 1
7	0350·4	...	...	...	...	100	116	140	160	40	− 45
8	0480·0	162	116	78	72	83	109	134	150	43	− 37
9	0609·6	147	136	114	86	90	...	...	...	36	− 30
10	0739·2	...	...	...	67	80	127	159	170	50	− 19
11	0868·8	181	183	178	146	114	107	125	147	40	+ 41
12	0998·4	151	148	135	100	74	89	...	...	36	− 2
13	1128·0	...	...	...	149	146	136	186	128	3	− 68
14	1257·6	150	165	148	128	128	140	164	184	22	− 26
15	1387·2	194	192	(181)	(160)	...	...	...	98	36	+ 61
16	1516·8	134	160	160	147	125	113	127	146	20	+ 58
17	1646·4	166	190	192	140	80	42	58	98	80	+ 58
18	1776·0	...	...	...	...	(96)	(76)	(80)	(115)	60	+ 58
19	1905·6	155	182	180	132	80	58	75	102	63	+ 61
20	2035·2	152	200	210	197	120	77	90	...	68	+ 77
21	2164·8	...	...	...	...	64	30	70	130	90	+ 58
22	2294·4	187	201	203	197	133	86	88	108	65	+ 75
23	2424·0	135	155	144	120	95	...	...	...	60	+ 63
24	2553·6	...	...	(192)	(147)	85	30	17	62	82	+ 79
25	2683·2	106	153	188	192	178	130	83	85	60	+123
26	2812·8	118	145	149	...	...	...	...	80	52	+134
27	2942·4	90	139	185	195	199	149	83	82	65	+138
28	3072·0	113	150	162	152	125	90	84	90	42	+ 94
29	3201·6	100	135	176	...	...	...	...	(89)	42	+128
30	3331·2	125	166	185	191	185	159	130	106	41	+134
31	3460·8	100	130	152	158	134	90	84	95	39	+110

6. If we calculate the 2nd and 3rd harmonics and reduce the three sets to standard form we get—

$$33\cos(\theta-180^\circ)+1\cdot5\cos(2\theta-182^\circ)+1\cdot0\cos(3\theta-352^\circ)$$
$$63\cos(\theta-180^\circ)+5\cdot0\cos(2\theta-119^\circ)+3\cdot0\cos(3\theta-47^\circ)$$
$$50\cos(\theta-180^\circ)+2\cdot2\cos(2\theta-123^\circ)+1\cdot1\cos(3\theta-341^\circ)$$

The coefficients of the 2nd and 3rd terms are very small, but the phases are consistent, and those of 2θ all indicate a Group II. variation, whereas the B.A.A. observations indicated Group I. (*M.N.*, **80**, 43), the corresponding expression being—

$$0{\cdot}38\cos(\theta-180^\circ)+0{\cdot}23\cos(2\theta-41^\circ)+0{\cdot}18\cos(3\theta-183^\circ).$$

The weighted mean value from Sawyer's observations is

$$47\cos(\theta-180^\circ)+2{\cdot}4\cos(2\theta-133^\circ)+1{\cdot}5\cos(3\theta-23^\circ).$$

7. As regards epoch, on p. 61 of *M.N.*, **80** a diagrammatic ephemeris is given for 1944 days, beginning with J.D. 2417704. Carrying this back $4\times1944=7776$ days, we get 2409928. But a rough discussion of observed maxima suggested (see p. 63) that we should subtract 96 days further for "halts," which brings us to 2409832, the initial date of the second cycle in Table II. Thus the accordance of the phase -12° or 348° with the 336° of the former paper is good, and justifies our period of 129·6 days very closely if the "halts" are correct. We have found two halts: after E = 14 for 77° or 28 days, and after E = 24 for 60° or 22 days, making together 50 days and leaving 46 more to be taken between E = 31 and E = 63. On p. 63 (*loc. cit.*) only the halt at E = 14 is confirmed.

8. The assumption was now made that the θ variation in each of the above sets was adequately represented by the mean values tabulated in § 5, which were accordingly subtracted from the individual constituents: and these were next arranged in lengths of 15 to study the ϕ variation. It was again quite easy to fill in gaps by taking the means of residuals in the preceding and following cycles. The deduced first harmonics for the 16 cycles were as follows:—

TABLE III.

No.	R sin ϕ_0.	R cos ϕ_0.	R.	ϕ_0.	No.	Means of Sets. α.	β.	γ.	δ.
1	− 2	+13	13	353°	1	+19	+13	−12	+21
2	− 6	+12	13	332	2	+17	+ 3	−29	+14
3	− 5	+17	18	343	3	+ 8	+ 3	−52	+12
4	− 6	+ 8	10	323	4	− 3	−10	−40	+19
5	−22	+17	29	308	5	− 9	−17	− 7	+33
6	−10	+19	22	332	6	−22	−26	− 3	+37
7	− 5	+14	15	342	7	−16	−21	+ 1	+16
8	−14	+24	28	327	8	−13	−14	+ 6	− 7
9	−11	+11	16	314	9	− 7	− 9	+11	−15
10	−20	+ 8	22	293	10	+ 4	− 1	+15	−19
11	−28	+16	32	301	11	+ 1	+10	+ 5	−21
12	−19	+ 9	21	296	12	+ 4	+20	−12	−32
13	−10	−20	23	206	13	+ 7	+24	− 6	−36
14	−19	−18	26	228	14	+13	+26	+ 2	− 9
15	− 8	− 7	10	227	15	+19	+16	0	+ 5
16	+28	+ 6	28	77					

Column 1 gives the number of the ϕ cycle and columns 2 and 3 the values of the coefficients of $\sin\phi$ and $\cos\phi$ in the expression $R\sin(\phi-\phi_0)$, the deduced values of R and ϕ_0 being added. The last five columns will be explained in a moment.

9. Taking the epochs of change in θ as occurring after the 13th and 23rd cyles of θ (E = 14 and 24), or after $13\times8=104$ and $23\times8=184$ terms, these points are approximately at the ends of the 7th and 12th cycles of ϕ. Now the second point has a well-marked change in ϕ of about 80°; but at the first the change is more doubtful. Without the suggestion from θ it would perhaps have escaped notice, or been taken lower down. But since the evidence of the B.A.A. observations was apparently in favour of discontinuities in θ and ϕ occurring simultaneously, and since there is apparently a simultaneous change at about the 180th term in the present series, we may provisionally accept a simultaneous change at the 105th term also. Hence we may group together the first seven cycles of ϕ, and we then obtain the means given under heading α in Table III.; then the next five cycles given under heading β; then the next three under heading γ; and finally the last cycle alone under heading δ. This is clearly quite different for ϕ, though Table II. suggests no special difference for θ, and the anomaly may be a mere accident.

10. Harmonic analysis of the sets gives the expressions

$$\begin{aligned}
\alpha &= 17\cos(\phi-330^\circ)+\ 7\cos(2\phi-\ 42)+\ 1\cos(3\phi-270^\circ)\\
\beta &= 22\cos(\phi-307^\circ)+\ 2\cos(2\phi-147)+\ 3\cos(3\phi-157^\circ)\\
\gamma &= 19\cos(\phi-215^\circ)+12\cos(2\phi-312)+12\cos(3\phi-308^\circ)\\
\delta &= 28\cos(\phi-\ 77^\circ)+\ 6\cos(2\phi-302)+12\cos(3\phi-335^\circ)
\end{aligned}$$

or, reducing to standard phase,

$$\begin{aligned}
\alpha &= 17\cos(\phi-180^\circ)+\ 7\cos(2\phi-102^\circ)+\ 1\cos(3\phi-180^\circ)\\
\beta &= 22\cos(\phi-180^\circ)+\ 2\cos(2\phi-253^\circ)+\ 3\cos(3\phi-136^\circ)\\
\gamma &= 19\cos(\phi-180^\circ)+12\cos(2\phi-242^\circ)+12\cos(3\phi-203^\circ)\\
\delta &= 28\cos(\phi-180^\circ)+\ 6\cos(2\phi-148^\circ)+12\cos(3\phi-284^\circ)
\end{aligned}$$

The phases of 2ϕ are all between 90° and 270°, which assigns the ϕ variation to Group II. as before, but they deviate considerably from the mean value (about 140°) for this group, as shown on pp. 376-7 of *M.N.*, 79. The coefficients are, however, all extremely small and doubtless affected by large accidental errors. If we take the weighted mean according to number of cycles, we get

$$20\cos(\phi-180^\circ)+2\cos(7\phi-163^\circ)+3\cos(3\phi-198^\circ),$$

which we may compare with

$$0{\cdot}26\cos(\phi-180^\circ)+0{\cdot}02\cos(2\phi-238^\circ)+0{\cdot}04\cos(3\phi-270^\circ)$$

given on p. 57 of *M.N.*, 80. Since our unit of one-tenth of a step is approximately one-hundredth of a magnitude, the coefficients are in good accord.

11. Returning to the unreduced expressions, we see that the phases of ϕ *diminish* instead of *increasing*, like those of θ in § 5, so that even if the changes are simultaneous they cannot be expressed in days counted for both θ and ϕ, as was adopted for the B.A.A. observations. Another point is that the changes in ϕ are unequal. The hypothesis of equality has worked so well that it seems desirable to retain it as a working hypothesis until some clear contradiction is encountered. In this case, if we take sets α and β together, the mean phase is 311°; and the three phases 311°, 215°, and 77° are separated approximately by two intervals of 120°, being not far from 323°, 203°, and 83°. The residuals of the B.A.A. observations were re-examined to see whether anything corresponding to this difference of behaviour in ϕ could be traced, but without success.

12. Another inference from the former discussion is not confirmed by Sawyer's observations. It was hazarded that the θ variation was waxing and ϕ waning. It is difficult to test the cases separately, as the value of a "step" is uncertain: but at least the ratio of the θ coefficient to that of ϕ should be increasing. Now for Sawyer's observations this ratio is $47/20 = 2{\cdot}35$; for the B.A.A. observations $38/26 = 1{\cdot}46$. Instead of the suggested increase therefore we get a decrease. Of course there may be fluctuations, but at present at any rate this increase of θ at the expense of ϕ, which led to some rather successful predictions about changes of period in stars of Groups I. and II., is not supported.

13. There was a third variation in about 15 cycles of θ or 8 of ϕ, which was tentatively identified as $2\theta - \phi = \chi$. For investigating this period we may take the mean of each row of Table II., and may first remove the θ variation. But we are debarred from subtracting mean values of ϕ, since the last cycle of ϕ stands alone. To eliminate the effect of ϕ as far as possible we may, however, combine each item (mean of 8 terms) with the mean of the preceding and following items, which are affected by nearly opposite phases of ϕ; and we then get two sets of 15 terms, which should represent χ:

$$-7 - 9 - 10 - 11 - 12 - 8 - 7 - \quad 6 + 3 + 11 + 16 + 24 + 16 + 12 + 5$$
$$\quad 0 \quad 0 \quad\; 0 + \; 3 + \; 2 + 3 - 5 - 10 - 7 - \; 8 - \; 6 - \; 7 - \; 2 + \; 4 - 17$$

There is no real resemblance between the two sets, and though the mean would give a sinuous curve, the existence of χ could not be established from this series alone. Accepting the means for what they are worth, harmonic analysis gives

$$6\cos(\chi - 270^\circ) + 4\cos(2\chi - 180^\circ),$$

the 3rd harmonic being sensibly zero.

The former expression was

$$8\cos(\chi - 192^\circ) + 9\cos(2\chi - 0^\circ) + 6\cos(3\chi - 270^\circ).$$

14. In noting these discrepancies, however, we may reasonably recall that Sawyer's observations are those of a single observer only, and we may well be content that the θ and ϕ variations are so clearly shown, leaving the smaller matters to the elucidation of future observations.

Summary.

(*a*) Ten years' observations of W Cygni (1885--95) by E. F. Sawyer confirm the period of 129·6 days or θ variation formerly found from B.A.A. observations (*M.N.*, **80**, 62). But they make it a Group II. variation, not Group I. as before.

(*b*) They also confirm the period of 243 days or ϕ variation, which is Group II. as before.

(*c*) They also confirm the occurrence of sudden changes of phase in both θ and ϕ, which *may* be always simultaneous in both, though the evidence is here not quite so clear. But the changes of phase of ϕ are in the opposite direction to those of θ, and of its own changes in the B.A.A. series.

(*d*) The $\chi = 2\theta - \phi$ variation is only doubtfully shown. It does not repeat in the second available cycle.

(*e*) Finally, the suggested waxing of the θ variation as compared with ϕ is not confirmed—the evidence is distinctly the other way; though, of course, there may possibly be fluctuations.

Micrometrical Measures of Wide Double Stars.

By W. S. Franks.

The following measures form the eighth annual list of double stars which have been measured at Brockhurst Observatory with the $6\frac{1}{8}$-inch Cooke equatorial. As regards the very wide doubles, the main object has been to furnish material for deducing proper motions. As may be seen by an inspection of Burnham's *General Catalogue*, the proper motions derived from differential micrometer measures are usually more reliable than those deduced from meridian observations, which are often discordant *inter se*. It will be seen by the "Notes" that the pairs here given are all in motion; except a few of *h*'s objects, and several anonymous pairs, of which these are the only measures. A brief list of other doubles measured, which do not show any motion, is given merely for reference—following the "Notes."

Abbreviations.—Arg, *Argelander*; A.G., *Astronomische Gesellschaft*; β, *Burnham*; Es, *Espin*; Fr, *Franks*; *h*, *Sir J. Herschel*; Ho, *Hough*; OΣ, *Otto Struve*; OΣΣ, *Poulkova Catalogue*, part ii.; Schj, *Schjellerup*; S, *Sir J. South*; Σ, *W. Struve, Mensuræ Micrometricæ*; Σ *rej.*, *Catalogus novus*; ang. *angle*; dist. *distance*; c.p.m., *common proper motion.*

Micrometrical Measures.

Star.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Ang.	Dist.	Nights.	Date.
	h m	° ′		°	″		
OΣΣ 256	0 2·8	+30 49	7·0, 7·2	115·0	106·74	3	1919·776
Arg I	,, 3·8	59 5	8·8, 8·8	145·6	23·95	3	·791
A.G. I	,, 12·8	8 56	8·5, 8·8	209·7	10·65	3	·800
h 1959	,, 17·4	21 6	Fr 9·0, 10·0	287·4	25·79	3	·800
Arg (Es)	,, 41·6	58 45	9·0, 9·0	79·1	48·49	2	·875
OΣΣ 9	,, 44·3	29 54	7·0, 8·0	237·3	99·38	3	·878
OΣΣ 11	1 1·6	38 6	7·2, 8·2	160·1	60·85	3	·878
A.G. 18	,, 22 9	13 57	8·6, 10·0	41·4	75·28	2	·950
Arg (Es)	,, 51·4	59 8	7·1, 9·2	219·3	44·85	2	1920·020
Σ 215	2 2·9	40 19	Fr 8·0, 9·5	59·5	20·35	2	·008
OΣΣ 24	,, 5·8	56 45	7·0, 8·0	331·1	67·00	3	·023
h 658	,, 49·0	9 22	Fr 8·5, 9·2	26·4	17·76	2	·040
OΣΣ 38	3 38·7	27 34	6·2, 6·3	43·2	126·67	3	·048
Σ 506	4 11·0	70 16	9·0, 9·2	294·8	9·58	2	·070
h 1142	,, 15·0	68 59	Fr 9·0, 9·4	81·8	24·21	3	·097
Σ 651	5 5·2	− 7 11	8·0, 9·5	39·6	24·54	2	·130
Σ 675	,, 11·2	− 5 42	8·8, 9·0	8·7	9·39	2	·130
Σ 679	,, 13·6	+25 4	8·7, 9·0	315·5	19·03	2	·130
Σ 683	,, 14·2	25 4	7·8, 10·0	77·4	12·20	2	·130
Σ 690	,, 18·2	57 44	8·5, 9·5	12·1	20·38	2	·132
Σ 733	,, 27·4	15 53	8·7, 9·5	41·7	12·46	2	·138
Σ 827 *rej.*	,, 55·0	− 0 31	8·0, 10·1	222·1	23·51	3	·149
Σ 902	6 20·8	+35 1	8·4, 9·4	151·6	11·90	2	·150
Σ 916	,, 25·7	56 43	8·5, 9·8	255·1	9·14	2	·150
Σ 974	,, 46·2	38 59	6·7, 10·0	225·1	22·99	2	·155
OΣΣ 83	7 3·4	25 55	6·0, 7·0	84·1	111·47	3	·160
Σ 1071	,, 15·8	45 11	8·3, 10·2	17·0	17·80	2	·203
OΣ 179	,, 38·4	24 38	3·7, 8·5	240·1	6·84	3	·211
OΣΣ 91	8 13·9	35 23	6·5 7·5	219·9	93·79	3	·211
Σ 1285	,, 45·6	21 16	Fr 9·2, 9·7	337·3	24·89	2	·215
h 115	,, 58·7	14 41	,, 8·8, 9·6	124·0	26·56	3	·218
OΣΣ 102	9 29·8	14 32	7·7, 8·5	43·2	48·22	3	·230
Σ 1453	10 31·9	−13 1	8·5, 9·7	231·4	9·31	2	·238
Σ 1462	,, 36·8	+51 19	7·8, 9·7	171·8	7·60	2	·268
OΣΣ 107	11 5·2	66 35	7·2, 7·5	44·9	72·04	3	·277
OΣΣ 108	,, 7·3	36 23	6·3, 7·0	69·8	138·40	3	·287
Σ 1524	,, 13·1	33 38	3·8, 10·0	146·2	7·30	2	·282
Σ 1648	12 25·5	4 4	7·8, 9·5	43·7	7·74	2	·290
Σ 1731	13 8·1	− 2 2	7·7, 9·8	302·3	8·83	2	·297

Micrometical Measures—continued.

Star.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Ang.	Dist.	Nights.	Date.
	h m	° ′		°	″		
Σ 1743	13 20·1	− 7 4	8·5, 8·8	79·7	6·07	2	1920·321
S 649	,, 25·1	+60 28	Fr 6·0, 8·0	110·6	182·75	2	·304
OΣΣ 124	,, 26·7	24 44	,, 7·0, 7·2	257·4	71·41	3	·313
Σ 1769	,, 33·6	39 41	7·3, 8·5	259·0	56·09	2	·310
S 655	,, 45·6	18 12	Fr 8·5, 9·5	81·2	40·23	2	·321
OΣΣ 127	,, 48·7	68 49	6·5, 8·0	64·3	78·21	2	·304
h 2746	14 38·8	70 10	9·0, 10·0	239·7	21·54	2	·335
Σ 1928	15 9·4	72 50	8·5, 9·2	275·2	6·60	2	·341
OΣΣ 137	,, 12·7	51 19	6·5, 8·5	103·8	72·90	3	·334
Σ 2006	,, 58·4	59 13	7·5, 8·0	216·9	49·09	2	·356
S 685	17 11·4	−26 32	Fr 7·0, 8·5	337·6	6·15	2	·427
OΣΣ 171	18 29·5	+38 45	6·5, 7·5	321·7	143·22	3	·388
Σ 2463	19 3·1	45 40	8·5, 10·2	4·6	9·47	2	·448
OΣΣ 177	,, 8·0	16 41	6·7, 7·5	284·8	113·47	3	·460
OΣΣ 181	,, 16·0	26 28	6·2, 6 3	182·8	57·36	3	·460
OΣΣ 207	20 19·6	42 40	6·5, 8·0	63·2	93·64	3	·503
h 1529	,, 28·0	− 6 34	7·8, 10·2	107·2	37·18	2	·532
OΣΣ 211	,, 46·8	+58 22	6·5, 7·0	263·0	109·53	3	·503
h 3033	21 27·5	6 21	Fr 8·8, 10·0	244·5	25·38	2	·616
Anon (Es)	,, 35·2	61 38	9·2, 9·3	18·9	33·61	2	·764
OΣΣ 224	,, 41·0	15 17	7·5, 8·5	3·9	58·38	3	·624
OΣΣ 228	,, 56·8	4 18	7·5, 9·0	26·1	76·62	3	·624
h 3114	22 22·4	−17 47	7·8, 9·0	94·1	6·87	2	·660
h 3149	,, 48·1	+4 8	Fr 9·0, 9·5	240·5	34·88	2	·673
OΣΣ 239	,, 50·5	35 49	5·5, 8·0	243·5	51·15	3	·704
Anon (Es)	,, 58·0	63 16	8·8, 9·2	114·3	44·10	2	·764
h 978	23 1·5	− 4 46	8·5, 10·0	290·4	15·23	2	·755
h 1864	,, 11·5	+42 6	Fr 9·2, 9·5	207·9	21·78	2	·764
OΣΣ 244	,, 15·0	47 49	6·3, 9·0	299·6	85·76	2	·772
OΣΣ 246	,, 23·0	23 2	7·5, 8·0	114·7	85·94	2	·772
h 1894	,, 31·0	50 59	Fr 9·6, 9·6	24·0	18·71	2	772
OΣΣ 251	,, 48·5	50 58	7·0, 9·0	201·4	44·14	2	·772

Notes.

OΣΣ 256. Ang. decreasing and dist. increasing; probable rectilinear motion.
Arg 1. Ang. and dist. increasing; only one previous measure.
A.G. 1. Ang. and dist. decreasing rapidly.
Arg (Es) B.D. +58°102–4; ang. and dist. decreasing; probable rectilinear motion.
OΣΣ 9. Ang. and dist. increasing, from the considerable proper motion of A.

OΣΣ 11. Ang. increasing and dist. decreasing, through proper motion of one of the stars.
A.G. 18. Dist. increasing; probable rectilinear motion.
Σ 215. Dist. slowly increasing in P.A. 59°.
OΣΣ 24. Dist. increasing in P.A. 331°, through proper motion of one or both stars.
h 658. Ang. increasing and dist. decreasing.
OΣΣ 38. Ang. and dist. increasing; A and B each having proper motion.
Σ 651. Ang. decreasing and dist. increasing; rapid rectilinear motion.
Σ 675. Ang. increasing; dist. constant.
Σ 679. Dist. slowly decreasing; rectilinear motion.
Σ 683. Ang. decreasing; dist. constant.
Σ 690. Ang. and dist. increasing; only two previous measures.
Σ 733. Ang. increasing. Σ 902. Small increase in ang.; dist. constant.
Σ 916. Ang. increasing; dist. fixed. Σ 974. 59 Aurigæ; probable c.p.m.
OΣΣ 83. Ang. decreasing and dist. increasing, through proper motion of both stars.
Σ 1071. Ang. and dist. increasing; rectilinear motion. OΣ 179. κ Geminorum.
OΣΣ 91. Certainly a change, though small; ang. decreasing and dist. increasing.
Σ 1285. Dist. slowly decreasing; rectilinear motion.
OΣΣ 102. Ang. increasing and dist. decreasing. Σ 1453. Small increase in ang. and dist.
Σ 1462. Small decrease in ang. and dist.
OΣΣ 107. Ang. and dist. increasing, due to the considerable proper motion of A in P.A. 250°.
OΣΣ 108. Ang. decreasing and dist. increasing, through the proper motion of A.
Σ 1524. ν Urs. Maj; c.p.m. Σ 1648. Ang. slowly increasing; dist. constant.
Σ 1731. Slow increase in ang.; dist. constant. Σ 1743. Ang. increasing.
S 649. Small increase in dist., due to proper motion of A; B forms Σ 1752.
OΣΣ 124. Ang. decreasing and dist. increasing; both stars having distinct proper motions.
Σ 1769, AC. Relatively fixed, but c.p.m. in the triple.
S 655. Ang. and dist. increasing, through proper motion of A.
OΣΣ 127. Ang. decreasing and dist. increasing, from proper motion of both stars.
Σ 1928. Ang. slowly decreasing, from precession. OΣΣ 137. Ang. and dist. decreasing; proper motion.
Σ 2006, AC. Ang. decreasing, dist. increasing; rectilinear motion; AB are physically connected.
S 685. 38 Oph. OΣΣ 171. Ang. and dist. increasing; proper motion in both A and B.
Σ 2463. Ang. decreasing; dist. constant.
OΣΣ 177. Ang. and dist. decreasing; proper motion in both stars; A forms β 139.
OΣΣ 181. Ang. decreasing and dist. increasing; proper motion.
OΣΣ 207. Dist. decreasing through proper motion of A, which is Ho 128.
h 1529. Ang. probably decreasing. OΣΣ 211. Ang. increasing, dist. decreasing; proper motion in A.
OΣΣ 224. Ang. decreasing; dist. constant. OΣΣ 228. Dist. increasing through proper motion of A.
h 3149. Ang. increasing—or error of 10° by *h* (230°·4)?
OΣΣ 239. Small increase in ang. and dist. *h* 978. Small increase in ang. and dist.
OΣΣ 244. Ang. decreasing and dist. increasing, through proper motion of A.
OΣΣ 246. Ang. increasing and dist. decreasing; both stars having proper motion.
OΣΣ 251. Ang. and dist. increasing, from proper motion of A.

List of other Pairs measured.

The following pairs were also measured between 1919·770 and 1920·772, but as no movement was indicated since the last previous measures it was deemed unnecessary to occupy space by giving them *in extenso*:—

h 1935, *h* 1944, Es 3, *h* 2010, *h* 2030, Σ 111, A.G. 16, A.G. 17, *h* 2052, Arg 5, Σ 154, *h* 2089, S 412, *h* 2143, Σ 304 *rej.*, Σ 309 *rej.*, *h* 2167, A.G. 67, Σ 433 *rej.*, A.G. 73, *h* 1140, Σ 487, Σ 482, A.G. 78, *h* 2234, Σ 686, Σ 756 *rej.*, Σ 768 *rej.*, *h* 783, *h* 713, Σ 871, Σ 901, Σ 913, Σ 953, Σ 959, Σ 970, S 537, Σ 988, Σ 992, Σ 995, Σ 1027, Σ 1020, Σ 1060, Σ 1080, Σ 1098, Σ 1120, Σ 1121, Weisse 16, Σ 1228, Σ 1246, Σ 1272, Σ 1274, *h* 111, Σ 1301, Σ 1320, Σ 3122, Weisse 24, Σ 1461, Σ 1503, Σ 1511, Σ 3068, Σ 3069 *rej.*, Σ 3074, Σ 1597 *rej.*, Σ 1610, Σ 1609, Σ 1712, Σ 1718, Σ 1729, Σ 1737, Σ 1790, Σ 1806, Σ 1811 *rej.*, Σ 1815, Σ 1829, Σ 1831, Σ 1859 *rej.*, *h* 561, Σ 1949, Σ 1960, Σ 1964, Σ 1968, *h* 577, Σ 3103 *rej.*, Σ 3104, Σ 2064 *rej.*, Σ 2070 *rej.*, O. Stone 34, *h* 590, Σ 2175, *h* 1303, Σ 2228, S 698, Σ 3129, Σ 2392, Σ 2395, OΣ 525, Σ 2418 *rej.*, *h* 1358, Σ 2460, OΣ 372, Σ 2528, Σ 2557, OΣ 390, H. C. Wilson 18, Es 132, *h* 607 (*a'* Capr.), *h* 2973, OΣ 410, Σ 2719 *rej.*, OΣ 417, OΣ 424, *h* 275, Σ 2788 *rej.*, Schj. 29, Σ 2805 *rej.*, *h* 1661, Σ 2818 *rej.*, Σ 2821 *rej.*, *h* 1786, *h* 3150, *h* 1841, *h* 1857, *h* 1862, Σ 2999 *rej.*, and several anonymous pairs.

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Note on the Sobral Eclipse Photographs.
By Henry Norris Russell.

1. The displacements of the star images on the photographs obtained during the eclipse of 1919 May 29* have given rise to much discussion, particularly since the observed shifts of some of the stars are not directed radially away from the Sun's centre. The present note deals with the reality of these "non-radial displacements" and with a possible explanation for them.

The observations at issue are those obtained with the 4-inch object glass at Sobral by Dr. Crommelin and Mr. Davidson. The co-ordinates of the star images on the eclipse plates were measured relatively to a "scale plate," and compared with those on an equal number of comparison plates, taken some six weeks later, under instrumental conditions as similar as possible.

The differences of the co-ordinates obtained in the two cases are the real basis of the discussion, the scale plate serving only as an intermediary. These differences are the combined effects

(*a*) of the differences in scale-value, orientation, and centering between the plates;

(*b*) of the distortion of the star-field at the time of eclipse, produced by the gravitational deflection of light;

* "A Determination of the Deflection of Light by the Sun's Gravitational Field," by F. W. Dyson, A. S. Eddington, and C. Davidson, *Phil. Trans.*, A, 220, 291-333, 1920. *Memoirs R.A.S.*, 62, Appendix. Referred to hereinafter as the "Report."

(*c*) of any other causes which may have operated to distort the field at the time of eclipse relatively to that six weeks later; and

(*d*) of the accidental errors of observation.

Of these influences (*a*) and (*d*) must have been present; and the observations prove the existence of (*b*). The differences in refraction at the somewhat different altitudes at which the eclipse and comparison plates were taken give rise to an apparent distortion, (*c*).

2. The refraction may be expressed as a series proceeding by powers of the co-ordinates x, y of the star images on the plate. The terms of the first order in x, y are given in the Report (p. 17).

Those of the second order are not mentioned. Calculation by Turner's formulæ * gives the following results, in which x and y are expressed in units corresponding to 50′ of arc :—

Eclipse plates :

$$\Delta^2 x = -0''{\cdot}031x^2 - 0''{\cdot}016xy - 0''{\cdot}013y^2$$
$$\Delta^2 y = -0''{\cdot}011x^2 - 0''{\cdot}016xy - 0''{\cdot}013y^2.$$

Mean of comparison plates :

$$\Delta^2 x = -0''{\cdot}052x^2 - 0''{\cdot}025xy - 0''{\cdot}018y^2$$
$$\Delta^2 y = -0''{\cdot}016x^2 - 0''{\cdot}022xy - 0''{\cdot}014y^2.$$

Only the difference between these two is of importance. The amount of this for the seven stars which were measured is :

TABLE I.

Eclipse minus Comparison Plates.

Star.	$\Delta^2 x$.	Diff.	$\Delta^2 y$.	Diff.
11	+0″·036	+0″·018	+0″·009	+″·004
5	+0·008	− ·010	·002	− ·003
4	+0·003	− ·015	·002	− ·003
3	+0·004	− ·014	·002	− ·003
6	+0·019	+ ·001	·007	+ ·002
10	+0·035	+ ·017	·013	+ ·008
2	+0·021	+ ·003	·004	− ·001

These quantities should be subtracted from the observed displacements of the stars on the eclipse plates. After subtracting the means for all the stars (which correspond only to minute differences of centering), the outstanding effects average only 0″·011 in x and 0″·003 in y, which justifies the neglect of them in the Report.

3. Whether there is any further distortion of the field can be determined only from the observations, after allowance for refraction.

* Allowance has been made for the misprint by which the longitude of Sobral is given 6^m too far west (Bauer, *Science*, **52**, 147, 1920).

For this purpose it is convenient to use the observed displacements of the individual star images on the eclipse and comparison plates given on p. 18. These have been corrected for differential refraction and aberration, and for the differences of centering, orientation, and scale between the plates. The latter corrections have been derived from the least-square solutions detailed on pp. 14-17, in which the plate constants were determined independently from the measures in x and y and the amount of the gravitational distortion was introduced as an additional unknown.

These "six-constant" solutions are equivalent to assuming that there is a real distortion of the field, expressible by a linear function of the co-ordinates—a "pure strain,"—in addition to differences in orientation and scale, which alone are assumed in a "four-constant" solution.

Transition to the latter was, however, made by freeing the plate constants from the effects of refraction and aberration, and then taking the mean of the two determinations of the orientation furnished by the measures in x and y on each plate, and of those of the scale-value derived from both co-ordinates on all the plates of certain groups, taken under presumably identical instrumental conditions. When reduced with these plate constants, and corrected for refraction and aberration, the measures give the residuals shown on p. 18 of the Report. Subtracting the results for the several comparison plates from those for the eclipse plates, the observed displacements are found. The means of these upon the seven available pairs of plates are given on p. 19, and may be regarded as the final data of observation for each star, while the residual differences of the individual plates from the mean measure the errors of observation.

It should be borne in mind, however, that, in the solutions for the plate constants and the deflection a, the unknowns are not independently determined, but are more or less "entangled," so that if a change is made in the derived (or assumed) value of a, the values of the other constants (especially the scale) must be changed too, in order to obtain the best "fit." This will be true to a less degree of any other change in the equations, such as the passage from six plate constants to four. Every different assumption of this sort, therefore, demands, theoretically at least, a separate adjustment of *all* the unknowns by least squares.

The extent to which any assumption corresponds with the observed data can then be assessed, by comparison of the sum of the squares of the outstanding residuals, with that which might be anticipated from the known mean error of the original observations. The latter can be found from the data of p. 18 by taking the differences between the eclipse plates and the corresponding comparison plates, and then subtracting the mean difference for each star from the seven individual differences, when we obtain residuals which can arise only from errors of observation. The mean-square

values of these residuals (mean errors) are 0″·230 for x and 0″·183 for y, or 0″·207 for both together, corresponding to a probable error of ±0″·140. The actual errors must be greater, for the tabular quantities are derived from solutions in which the plate constants have been adjusted to fit the observations. The whole number of such constants is 23 (14 expressing the differences of centering and 7 of orientation between the eclipse and comparison plates of each pair, and the other 2 the differences of scale-value between the eclipse plates as a whole and the two groups into which the comparison plates were divided). Add to these the 14 mean differences between the x's and y's of the images on the eclipse and comparison plates, and we have in all 37 constants which have been derived from the 98 observations.

The mean errors just found must therefore be multiplied by $\sqrt{\frac{98}{61}}$, which gives 0″·263 as the actual mean error of the measured difference of the co-ordinates on a pair of plates. The corresponding probable error is ±0″·177, or ±0″·125 for the measures of each plate separately, assuming them equally precise. In view of the unfavourable conditions of observation, this is a high degree of precision.

The distribution of the individual residuals in magnitude and sign is given in Table II. The last column gives the number which might have been anticipated on the theory of errors, with the probable error ±0″·140. The actual distribution agrees with this remarkably well.

TABLE II.

Residuals for Individual Images.

Limits.	R.A. Pos.	R.A. Neg.	Dec. Pos.	Dec. Neg.	All.	Theory.
0″·0 to 0″·1	8	12	8	8	36	36
0·1 to 0·2	5	5	11	10	31	29
0·2 to 0·3	8	5	2	4	19	18
0·3 to 0·4	1	1	3	3	8	9
0·4 to 0·5	1	...	...	...	1	4
0·5 to 0·6	1	1	...	...	2	1½
0·6 to 0·7	...	1	...	...	1	½
All	24	25	24	25	98	98

4. We may now proceed to discuss the mean results obtained from all the plates. Taking the observed displacements given on p. 19,* subtracting the amounts calculated on the basis of Einstein's theory, and also the second-order refraction corrections given in Table I., we obtain the quantities Δx and Δy which are given in Table III.

* Correcting an error in sign for the R.A. of star 6.

TABLE III.

Mean Results.

Star.	Δx.	V_1.	V_2.	V_3.	V_4.
11	+0″·11	+0″·13	+0″·09	+0″·05	+0″·02
5	+ ·03	+ ·02	+ ·06	− ·09	− ·05
4	·00	+ ·03	+ ·05	+ ·05	+ ·06
3	− ·07	− ·04	− ·01	− ·03	− ·01
2	− ·14	− ·09	− ·09	− ·02	− ·02
10	− ·19	− ·14	− ·12	− ·04	− ·04
6	+ ·10	+ ·10	+ ·02	+ ·09	+ ·03
Σv^2	·0836	·0575	·0372	·0241	·0095

Star.	Δy.	V_1.	V_2.	V_3.	V_4.
11	+0″·14	+0″·13	+0″·14	+0″·05	+0″·03
5	−0·03	− ·07	− ·03	+ ·03	+ ·05
4	+0·09	+ ·04	− ·03	+ ·02	− ·03
3	+0·13	+ ·08	− ·02	+ ·08	·00
2	+0·17	+ ·10	+ ·15	+ ·06	+ ·09
10	+0·02	− ·05	+ ·02	− ·08	− ·03
6	−0·18	− ·25	− ·22	− ·06	− ·04
Σv^2	·1077	·1048	·0931	·0238	·0149

It may now be assumed—

1. That the gravitational displacement has the theoretical value and that there is no other distortion of the field.
2. That there is no distortion of the field, but that the gravitational effect is to be found from the observations.
3. That the gravitational effect has its theoretical value, and that there is an additional distortion of the field, expressible as a linear function of the co-ordinates x and y.
4. That this distortion exists, while the gravitational displacement is to be found from the observations.

Least-square solutions have been made on these four hypotheses. The equations of condition in case 4 are

$$c + by + ax + \mathrm{E}_x a = \Delta x$$
$$f + ey + dx + \mathrm{E}_y a = \Delta y,$$

as on p. 14 of the Report, except that here a represents a correction to the theoretical gravitational displacement at 50′ from the Sun's centre, which is 0″·553. In cases 1 and 3 the terms in a are omitted, and in cases 1 and 2 we must set $a = e$, $b = -d$. The equations of condition for case 4, and all the normal and elimination equations, are given on p. 14 of the Report, so that the

solutions with the new observational terms were very easily made. The resulting values of the constants are given in Table IV.

TABLE IV.

Solution.	1.	2.	3.	4.	4x.	4y.
	″	″	″	″	″	
a	+0·004	−0·065	−0·028	−0·090	−0·120	..
b	− ·027	− ·018	− ·101	− ·081	− ·072	...
c	− ·017	− ·005	+ ·008	+ ·017	+ ·021	...
d	+ ·027	+ ·018	− ·126	− ·122	...	− ·123
e	+ ·004	− ·065	+ ·118	+ ·061	...	+ ·075
f	+ ·043	+ ·013	+ ·053	+ ·027	...	+ ·032
α	...	+ ·101	...	+ ·080	+ ·118	+ ·061

Solution 4 gave separate determinations of α from the x and y measures, which differed somewhat. The mean was taken, giving the value derived from y double weight, and this value of α was substituted in the normal equations, which were re-solved for the other constants. The results are given in column 4, and the separate solutions from x and y in columns 4x and 4y.

It is clear that every change in the hypotheses demands sensible changes in all the plate constants in order to get the best fit for the observations. In particular, it should be noted that the values of α and of the scale constants a and e are strongly correlated, a change in the former demanding alterations in the latter of opposite sign and about two-thirds as great. If the stars were all equidistant from the Sun, the gravitational displacement would obviously be indistinguishable from a certain change in scale; and, in fact, the changes in α and a nearly neutralise one another at the mean distance of the stars from the Sun, 55′. It may also be observed that the scale and orientation constants in the four-constant solutions are not equal to the means of the values derived from the x's and y's separately in the six-constant solution. This arises from the irregular distribution of the stars in the field.

5. Solution 4 is substantially a repetition of that in the Report, from which x and y are derived, and should reproduce its results. The mean of the corrections derived from the two co-ordinates should vanish for the orientation and scale. They are actually −0″·020 and −0″·014. The value of the gravitational distortion at distance 50′ is 0″·553 +0″·080, or 0″·633, as against 0″·100 or 0″·625 found in the Report (p. 16). The minute outstanding differences arise probably from "neglected decimals" in the long series of calculations.

The deflection at the Sun's limb comes out 2″·00 in solution 4 as against 1″·98 in the Report,—an agreement far within the probable error. The residuals obtained for each star image, for all four solutions, are given in Table III. The sums of their squares are given at the foot of each column, and collected in Table V.

Table V.

Solution.	Gravitational Deflection derived from	Distortion of Field.	Sum of Squares of Residuals. Observed.	Theory.
1	Theory	Absent	0·1623	0·0990
2	Observation	Absent	·1303	·0891
3	Theory	Present	·0479	·0792
4	Observation	Present	·0244	·0693

It is to these quantities that we must appeal to determine whether the eclipse field is actually distorted. Each of the 14 quantities Δx, Δy is the mean of 7 observed data. Let the mean error of one of these be ϵ; then the sum of the squares of the residuals from a solution in which m unknowns have been derived from the 14 equations of condition should be $\left(2-\frac{m}{7}\right)\epsilon^2$. Introducing $\epsilon = 0''\cdot263$, we find the values given in the last column of Table V.

Comparing the observed values of this sum with those which might be anticipated from the known accidental errors of observation, it appears that, unless the existence of distortion is assumed, the residuals are considerably larger than might have been expected. But when distortion is assumed the theoretical value of the gravitational deflection represents the mean result of the observations better than might have been expected from the mutual discordance of the individual plates. With the larger deflection derived directly from the observations, the agreement is better still—so much better, in fact, than there was any reason to anticipate, that a large part of the improvement must apparently be attributed to chance.

The existence of a distortion of the eclipse field appears to be proved. It has not been demonstrated that it follows the law assumed above, but only that a distortion of this sort, combined with the effects predicted by Einstein's theory, gives a sufficient representation of the observations.

Other forms of distortion might be invented which would do the same, but the simple linear form has the double advantage of being easy to compute, and easy to explain, as we shall soon see.

6. The amount of this distortion is represented by the quantities $a-e$ and $b+d$. The values obtained in solutions 3 and 4 are given below, with that derived from the Report (p. 17), by taking the differences of the scale value and orientation derived from the x and y measures for each plate, subtracting the means for the comparison plates from those for the eclipse plates, and multiplying the results by 6·25 to reduce to the units here employed.

	$a-e$.	$b+d$.
Solution 3	$-0''\cdot146\pm0''\cdot053$	$-0''\cdot227\pm0''\cdot053$
Solution 4	$-0\cdot151\pm0\cdot078$	$-0\cdot203\pm0\cdot055$
Report	$-0\cdot181$	$-0\cdot237$

The agreement of the three results shows that the determination of the distortion is very little entangled with that of the other unknowns.

The probable errors are computed from the weights of a, b, . . . etc., resulting from the least-square solutions, and the value of ϵ found above.

This distortion may be expressed as a contraction of all distances in the field measured parallel to a certain direction, leaving those at right angles to it unaltered. If the amount of the contraction at unit distance is S and the position angle p (measured from the y-axis toward the x-axis, so as to conform to the usual astronomical definition), we find easily

$$a - e = \mathrm{S} \cos 2p, \qquad b + d = -\mathrm{S} \sin 2p,$$

whence (taking the mean of solutions 3 and 4) $p = 62^\circ \pm 7^\circ$, $\mathrm{S} = 0''{\cdot}26 \pm 0''{\cdot}06$ at unit distance (50′).

This is a fairly large distortion, amounting to 1/12000 of the distance. For example, it is almost one-third of the whole differential refraction (of the first order) for the eclipse plates, which is expressible by $\mathrm{S} = 0''{\cdot}90$, $p = 60^\circ$. Though the position angle of greatest contraction is sensibly identical with the vertical, it is obviously far too great to be explained by any anomalies of terrestrial refraction. The fact that it increases with distance from the Sun is strongly against any hypotheses of "cosmic" origin (which in any case should be accepted only as a last resort).

7. The most probable source is in instrumental errors—and here the cœlostat mirror comes instantly under suspicion. The 16-inch mirror of the other cœlostat at Sobral showed serious astigmatism at all times (p. 8), and was deformed by the heat of the Sun to such a degree as almost to ruin the definition of the eclipse plates; and the 8-inch mirror, which fed the 4-inch object glass, gave images which "were not quite perfect" (p. 8).

Curvature of the mirror surface will damage the definition and also distort the field. Consider rays in any given plane passing through the axis of the telescope. Let the radius of curvature of the section of the mirror by this plane be R and the angle at which the axial ray meets this curve be $90 - \theta$, and let A be the aperture of the objective and D the distance from the objective to the mirror. Rays parallel to the axis from opposite sides of the objective will strike the mirror at points separated by the distance A sec θ. After reflection they will diverge (if this section of the surface is convex) by the angle $\frac{2\mathrm{A} \sec \theta}{\mathrm{R}}$ (in circular measure). This represents the angular diameter (measured in the position angle corresponding to the given plane) of a star image upon a plate which would be in sharp focus if the mirror was flat.

For a second star, at an angular distance ϕ from the axis, the diameter of the image will be the same (to the first order of small quantities). But the central ray in the case will meet the mirror at the distance $\phi \mathrm{D} \sec \theta$ from the point at which the axial ray

meets it, and the effective region of the mirror will be inclined as a whole at an angle $\frac{\phi D \sec\theta}{R}$ to the region which was operative in the first case. The second star image will therefore be not only distorted, but shifted bodily, by the amount $\frac{2\phi D \sec\theta}{R}$, inward if the mirror is convex in section.

This is a very well-known effect, familiar to anyone who has looked into a convex mirror. It increases with distance from the mirror, while the blurring of the focal images does not.

The analysis in two dimensions is equally simple. If the angle of incidence of the axial ray on the mirror is i, it is found that all distortions parallel to this plane are increased in the ratio sec i, while those perpendicular to it retain sensibly the same values as if the mirror was at normal incidence.

For the eclipse photographs the plate centre was in declination $+21^\circ{\cdot}7$ and hour angle $-39^\circ{\cdot}0$, while the normal to the cœlostat mirror was directed toward the equator in hour angle $-65^\circ{\cdot}2$. Hence the angle of incidence was $33^\circ{\cdot}5$, and the plane of incidence cut the sky at the plate centre in position angle $127^\circ{\cdot}0$ (in the usual notation) and the mirror along a line making an angle of 132°, measured clockwise on the upper face, with the north end of the polar axis. If the position angle of the maximum observed distortion was in or near 127°, we could explain it very simply by a spherical curvature of the mirror surface, which produces, with oblique incidence, an apparent astigmatism proportional to sec $i - 1$. But as the observed angle is 62°, this attractive explanation is unavailable, and we must assume that the mirror actually suffered an unsymmetrical distortion, presumably by heating by the Sun's rays. Since any arbitrary small change in the scale may be superposed on the observed distortion, and attributed to the figure of the mirror, an infinite variety of assumptions are possible regarding the latter. In general, the figure of the surface comes out ellipsoidal or hyperboloidal, but in two special cases it is cylindrical.

If the incidence on the mirror was normal, the axis of these cylinders would be parallel and perpendicular to the direction of greatest contraction of the field (one being concave and the other convex), and the radius of curvature would in both cases be given by $R/D = 2\phi/S = 6000''/0''{\cdot}26 = 23000$.

An exact calculation, taking into account the oblique incidence, shows that for the convex cylinder the generators are parallel to position angle 24° (measured clockwise from the north point of the mirror) and the radius $R = 24000D$, while the concave cylinder has generators in 118° and radius $27000D$.

The distance D is not stated in the Report. It must have been less than 1·5 metres, in order that the full aperture of the object glass be used for stars at the edge of the field, and was probably as much as 0·5 metre,* for reasons of convenience. Even with this

* [Approximately 1 metre.—F. W. D.]

smallest value, the radius of cylindrical curvature of the mirror, necessary to explain the observed distortion, is twelve kilometres. The corresponding elevation of the middle of the 8-inch mirror relative to the edges (or depression for the concave distortion) is 0·4 microns,—more generally $\frac{0\cdot2}{D}\mu$ when D is measured in metres. Such a distortion does not appear to be improbably great.

Indeed, there is clear evidence (p. 17) that, between July 13 and 14, the scale value of the plates altered abruptly by $0^{R}\cdot097$ (corresponding to $0''\cdot60$ in the present notation), although both sets of plates were taken at night, and no reference is made to any alteration in the instrumental adjustments; indeed, "the telescopes and cœlostats were left *in situ*" (p. 11), and "the breech end was securely screwed up to avoid any chance of subsequent movement" (p. 8).

This change can apparently be attributed only to a change in curvature of the mirror (substantially spherical, and more than twice as great as the cylindrical curvature necessary to account for the distortion at the time of eclipse). If such a change could happen between one night and the next, it is not improbable that the other might be caused by exposure to the direct rays of the Sun.

8. The concomitant distortion of the star images must, however, be remembered. With the value of R just found, the difference between the greatest and least diameter of the star images should be approximately A/12000D, or, reducing to arc and expressing D in metres, $\frac{1''\cdot7}{D}$. Nothing like this difference exists. Examination under the microscope of the images on one of the prints distributed by the Royal Observatory shows that they range from 0·06 to 0·15 mm. in diameter—that is, from $2''$ to $5''$,—and are apparently almost perfectly round, except for star 11, which is near the edge of the field, and shows an image elongated by about $1''\cdot5$. The average elongation of the images cannot, however, exceed $0''\cdot5$, and is probably less.

This is not, however, a disproof of the suspected astigmatism of the mirror, for there is a certain position in the astigmatic cone of rays, where the elliptical images are replaced by a "circle of least confusion," of diameter equal to half the difference of the axes of the ellipse. If the eclipse plates happened, by good fortune, to be near this position, the definition of the images would not be sensibly impaired by the amount of astigmatism here suggested; but the distortion of the field would nevertheless appear to its full extent.

9. The results of this investigation may be very briefly stated.

The photographs obtained at Sobral with the 4-inch objective show conclusive evidence that in addition to the gravitational displacement of the star images on the eclipse plates, there existed a distortion of the field equivalent to a contraction of all distances

parallel to the vertical by $\frac{1}{12000}$ of their amount (or a corresponding expansion of horizontal distances).

Unless such a distortion is admitted, the discordance between the mean results of the eclipse and comparison plates is much greater than would be anticipated from the mutual agreement of the results from individual pairs of plates. Admitting the distortion, the theoretical value of the "Einstein effect" represents the observations better than might have been anticipated; and the larger value derived from the observations gives *too* good an agreement, which appears to be mainly a matter of chance.

The observed distortion of the field can be explained by an alteration of the cœlostat mirror by the Sun's heat, introducing a cylindrical curvature of at least 12 kilometres radius. So minute a distortion is not improbable.

Princeton University Observatory:
1920 *November* 6.